# What is the Expected Return on a Stock?

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**Abstract**

We derive a formula that expresses the expected return on a stock in terms of the risk-neutral variance of the market and the stock's excess risk-neutral variance relative to the average stock. These components can be computed from index and stock option prices; the formula has no free parameters. We test the theory in-sample by running panel regressions of stock returns onto risk-neutral variances. The formula performs well at 6-month and 1-year forecasting horizons, and our predictors drive out beta, size, book-to-market, and momentum. Out-of-sample, we find that the formula outperforms a range of competitors in forecasting individual stock returns. Our results suggest that there is considerably more variation in expected returns, both over time and across stocks, than has previously been acknowledged.

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In this paper, we derive a new formula that expresses the expected return on an individual stock in terms of the risk-neutral variance of the market, the risk-neutral variance of the individual stock, and the value-weighted average of individual stocks' risk-neutral variance. Then we show that the formula performs well empirically.

The inputs to the formula—the three measures of risk-neutral variance—are computed directly from option prices. As a result, our approach has some distinctive features that separate it from more conventional approaches to the cross-section.

First, since it is based on current market prices rather than, say, accounting information, it can be implemented in real time (in principle; given the data available to us, we update the formula daily in our empirical work). Nor does it require us to use any historical information; it represents a parsimonious alternative to pooling data on many firm characteristics (as, for instance, in Lewellen, 2015).

Second, our approach provides conditional forecasts at the level of the individual stock: rather than asking, say, what the unconditional average expected return is on a portfolio of small value stocks, we can ask, what is the expected return on Apple, today?

Third, the formula makes specific, quantitative predictions about the relationship between expected returns and the three measures of risk-neutral variance; it does not require estimation of any parameters. This can be contrasted with factor models, in which both factor loadings and the factors themselves are estimated from the data (with all the associated concerns about data-snooping). There is a closer comparison with the CAPM, which makes a specific prediction about the relationship between expected returns and betas; but even the CAPM requires the forward-looking betas that come out of theory to be estimated based on historical data.

Our approach does not have this deficiency and, as we will show, it performs better empirically than the CAPM. But—like the CAPM—it requires us to take a stance on the conditionally expected return on the market. We do so by applying the results of Martin (2016), who argues that the risk-neutral variance of the market provides a lower bound on the equity premium. In fact, we exploit Martin's more aggressive claim that, empirically, the lower bound is approximately tight, so that risk-neutral variance directly measures the equity premium. We also present results that avoid any dependence on this claim, however, by forecasting expected returns *in excess of the market*. In doing so, we isolate the purely cross-sectional predictions of our framework that are independent from the "market-timing" problem of forecasting the equity premium.

We introduce the theoretical framework in Section 1; then we show how to construct the three risk-neutral variance measures, and discuss some of their properties, in Section 2.

Our central empirical results are presented in Section 3. We test the framework for S&P 100 and S&P 500 firms at the individual stock level using forecast horizons from one month to one year. It may be worth emphasizing that papers in the predictability literature typically set themselves the goal of identifying predictor variables that are statistically significant in forecasting regressions. We share this goal, of course, but since our model makes *ex ante* predictions about the quantitative relationship between expected returns and risk-neutral variances, we hope also to find that the estimated coefficients on the predictor variables are close to *specific numbers* that come out of our model. For most specifications we find that that we cannot reject the model, and we can reject the null hypothesis of no predictability at the six- and twelve-month horizons. Moreover, we do not reject our model when applied to the returns of portfolios sorted by beta, book-to-market, and past returns. We are able to reject the model on size-sorted portfolios, however: it turns out that the sensitivity of portfolio returns to stocks' risk-neutral variance is even stronger than our theory predicts. (This would be a victory for the conventional approach, which seeks only to establish statistical significance, but is a defeat for us.)

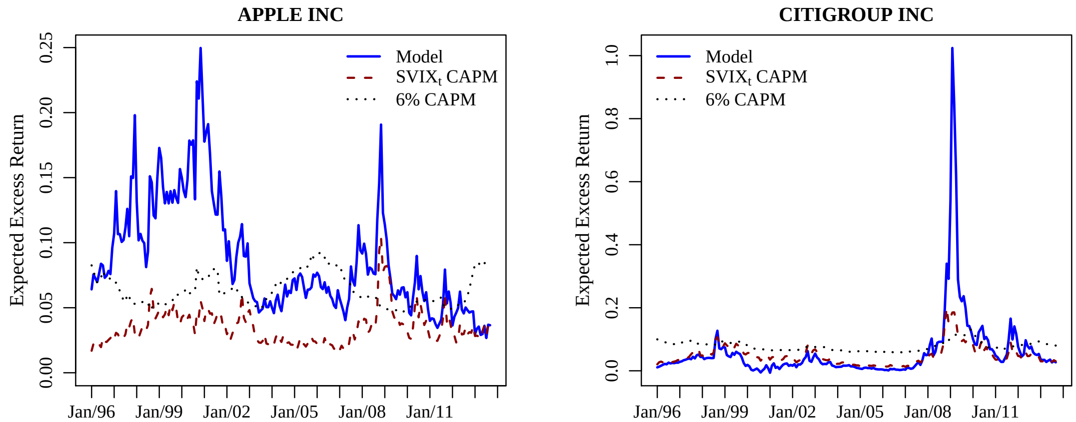
In Section 4, we investigate the relationship between stock characteristics and realized, expected, and unexpected (that is, realized minus expected) returns. We start by running panel regressions of each of the three onto beta, size, book-to-market, and past returns. We find no systematic relationship between characteristics and unexpected returns, and we do not reject the joint hypothesis that our risk-neutral variance predictive variable drives out the characteristics as a forecaster of returns and that it enters with a coefficient of 0.5, as our theory predicts. We also find that adjusted $R^2$ increases from about 1% using the characteristics alone to about 4% when risk-neutral stock variance is added. The evidence on whether characteristics help to predict *expected* returns is more mixed: the conclusions depend on whether we compute expected returns using the theory-implied coefficient of 0.5 (in which case the characteristics are strongly jointly significant) or whether we use the estimated coefficient that emerges from the panel regressions of Section 3 (in which case they are not). In any case, both possibilities are perfectly consistent with our theoretical results.

Section 5 assesses the out-of-sample predictive performance of the formula when its coefficients are constrained to equal the values implied by our theory. We compute out-of-sample $R^2$ coefficients that compare our formula's predictions to those of a range of competitors, as in Goyal and Welch (2008). We start by comparing against competitors that are themselves out-of-sample predictors. The formula outperforms all such competitors at horizons of 3, 6, and 12 months, both for expected returns and for expected returns in excess of the market. We go on to compare, more ambitiously, against competitors that have *in-sample* information. At the 6- and 12-month horizons, the only case in which our model 'loses' is when we allow the competitor predictor to know both the in-sample average (across stocks) realized return *and* the multivariate in-sample relationships between realized returns and beta, size, book-to-market, and past returns. (When we allow the competitor to know only the in-sample average and the univariate relationship between realized returns and any one of the characteristics, our formula outperforms.) In the purely cross-sectional case in which we forecast returns in excess of the market, the formula even outperforms the competitor armed with knowledge of the multivariate relationship. Given these successes, it is natural to ask how the resulting forecasts can be used in trading strategies. We show how to do so, and show also that the resulting strategies have attractive properties.

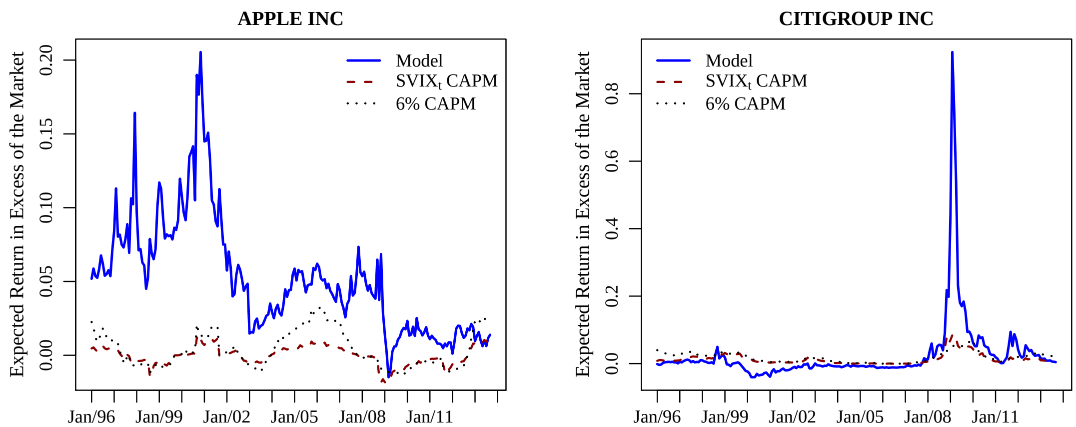
The empirical success of our formula is particularly striking because it makes some dramatic predictions about stock returns. Figure 1 plots the time-series of expected excess returns, relative to the riskless asset and relative to the market, for Apple and for Citigroup over the period from January 1996 to October 2014. According to our model, expected returns were extremely spiky for both stocks in the depths of the financial crisis of 2008–9. In the case of Apple, this largely reflected a high market-wide equity premium rather than an Apple-specific phenomenon. In sharp contrast, Citigroup's expected excess return at that time was far higher—by a factor of about five times, i.e. 80 percentage points—than the equity premium; for reference, Citigroup's CAPM beta at this time was around 1.8–1.9. The figure also plots expected excess returns computed using the CAPM with one-year rolling historical betas (and the equity premium computed from the SVIX index of Martin (2016), or fixed at 6%), to illustrate the point—which, as we will show, holds more generally—that our model generates far more volatility in expected returns, both over time and in the cross-

Figure 1: Expected excess returns and expected excess market returns. Annual horizon.

*Panel A. Expected excess returns*




*Panel B. Expected returns in excess of the market*




section, than does the CAPM.

*Related literature.* We believe our approach has two important advantages relative to recent work on the relation between volatility or idiosyncratic volatility and equity returns.[1] First, we exploit forward-looking information embedded in stock options, rather than relying on backward-looking measures of realized volatility calculated from historical equity return data. Second, our measures of stock variance are model-free,

[1] Papers in this literature typically focus on idiosyncratic volatility, defined as the volatility of a residual after controlling for systematic risk, whereas the volatility index $\text{SVIX}_{i,t}$ that plays a central role in this paper (and which is defined below) measures *total* stock-level volatility. We note, however, that total volatility measures based on historical returns are empirically similar to idiosyncratic volatility measures computed from factor model residuals; see for example Herskovic et al. (2016).

which is comforting in light of the simultaneous agreement, in the literature, about the informativeness of idiosyncratic volatility for future stock returns, and disagreement as to whether the predictive relationship is positive or negative. The source of this disagreement may be rooted in the measurement of idiosyncratic volatility. For instance, Ang et al. (2006) find a negative relation for total volatility as well as for idiosyncratic volatility defined as the residual variance of Fama–French three factor regressions on daily returns over the past month. By contrast, Fu (2009) finds a positive relation when idiosyncratic volatility is measured by the conditional variance obtained from fitting an EGARCH model to residuals of Fama–French regressions on monthly returns.

Our model attributes an important role to average stock variance (measured as the value-weighted sum of individual stock risk-neutral variances), a prediction that we confirm empirically. This result echoes the finding of Herskovic et al. (2016) that idiosyncratic volatility (measured from past returns) exhibits a strong factor structure and that firms' loadings on the common component predict equity returns. Furthermore, our measure of average stock variance may capture a potential factor structure in the cross-section of equity options, as documented by Christoffersen et al. (2015) across 29 Dow Jones firms.

Several authors have investigated whether options-based measures contain information for stock returns (see, for example, Driessen et al., 2009; Buss and Vilkov, 2012; Chang et al., 2012; Conrad et al., 2013; An et al., 2014). Two features distinguish our work from these studies. First, we develop a theory that derives the expected stock return as a function of risk-neutral variances only. This allows us to compute expected equity returns without any parameter estimation. Second, we operate on the level of individual stocks, rather than defining the cross-section in terms of portfolios. In other words, rather than asking whether options convey information for portfolio returns, we test a theory of expected returns directly at the level of the individual firm, thereby avoiding problems associated with asset pricing tests conducted with portfolios (see, e.g., Ang et al., 2010; Lewellen et al., 2010). Moreover, given that we can compute the expected return on a stock from current option prices, our results do not depend on the choice of a specific estimation window (see, for instance, Lewellen and Nagel, 2006, in the context of conditional CAPM tests).

In a more closely related paper, Kadan and Tang (2016) adapt an idea of Martin (2016) to derive a lower bound on expected stock returns. To understand the main differences between their approach and ours, recall that Martin starts from an identity that relates the equity premium to a risk-neutral variance term and a (real-world) covariance term; he exploits the identity by first arguing that a *negative correlation condition* (NCC) holds for the market return, i.e. that the covariance term on the right-hand side of this identity is nonpositive in all quantitatively reasonable models of financial markets. If so, the risk-neutral variance of the market provides a lower bound on the equity premium. Kadan and Tang (2016) modify this approach to derive a lower bound for expected stock returns based on a negative correlation condition for individual stocks. But it is trickier to make the argument that the NCC should hold at the individual stock level, so Kadan and Tang's approach only applies for a subset of S&P 500 stocks. In the present paper, we take a different line, exploiting the stronger claim in Martin (2016) that, empirically, the covariance term is approximately zero, so that risk-neutral market variance directly measures the equity premium. This more aggressive approach delivers a precise prediction for expected returns rather than a lower bound, and our hope is that it applies to all S&P 500 stocks.

# 1 Theory

Our starting point is the gross return with maximal expected log return: call it $R_{g,t+1}$, so $\mathbb{E}_t \log R_{g,t+1} \geq \mathbb{E}_t \log R_{i,t+1}$ for any gross return $R_{i,t+1}$. This *growth-optimal return* has the special property,[2] unique among returns, that $1/R_{g,t+1}$ is a stochastic discount factor. To see this, note that it is attained by choosing portfolio weights $\{g_n\}_{n=1}^N$ on the tradable assets[3] $\{R_{n,t+1}\}_{n=1}^N$ to solve

$$\max_{\{g_n\}_{n=1}^N} \mathbb{E} \log \sum_{n=1}^N g_n R_{n,t+1} \quad \text{such that} \sum_{n=1}^N g_n = 1.$$

[2]There is an analogy with the *minimal-second-moment return*, $R_{*,t+1}$, which has the special property that it is proportional to an SDF (Hansen and Richard, 1987), rather than inversely proportional as the growth-optimal return is. The minimal-second-moment return can be viewed as the theoretical foundation of the factor pricing literature; see Cochrane (2005) for a textbook treatment. In principle, given perfect data, either approach could be used to interpret asset prices, but we believe that our approach has an important practical advantage: if we started from $R_{*,t+1}$ rather than $R_{g,t+1}$, the analog of equation (3) would not let us exploit the information in option prices so straightforwardly.

[3]These include not only the stocks in the index, but also stock options.

The first-order conditions for this problem are that

$$\mathbb{E}\left(\frac{R_{i,t+1}}{\sum_{n=1}^{N} g_n R_{n,t+1}}\right) = \psi \quad \text{for all } i,$$

where $\psi$ is a Lagrange multiplier; we follow Roll (1973) and Long (1990) in assuming that these first-order conditions have an interior solution. Multiplying by $g_i$ and summing over $i$, we see that $\psi = 1$, and hence that the reciprocal of $R_{g,t+1} \equiv \sum_{n=1}^{N} g_n R_{n,t+1}$ is an SDF.

We can therefore calculate the time-$t$ price of a claim to the time-$(t+1)$ payoff $X_{t+1}$ in either of two ways. We could write, with reference to the growth-optimal return,

$$\text{time-}t \text{ price of a claim to } X_{t+1} = \mathbb{E}_t\left(\frac{X_{t+1}}{R_{g,t+1}}\right). \tag{1}$$

Alternatively, we could write

$$\text{time-}t \text{ price of a claim to } X_{t+1} = \frac{1}{R_{f,t+1}}\,\mathbb{E}_t^*\,X_{t+1} \tag{2}$$

where $\mathbb{E}_t^*$ is the risk-neutral expectation operator. In particular, if $X_{t+1} = R_{i,t+1}R_{g,t+1}$ is a tradable payoff[4] then (1) and (2) must agree, and we can conclude that

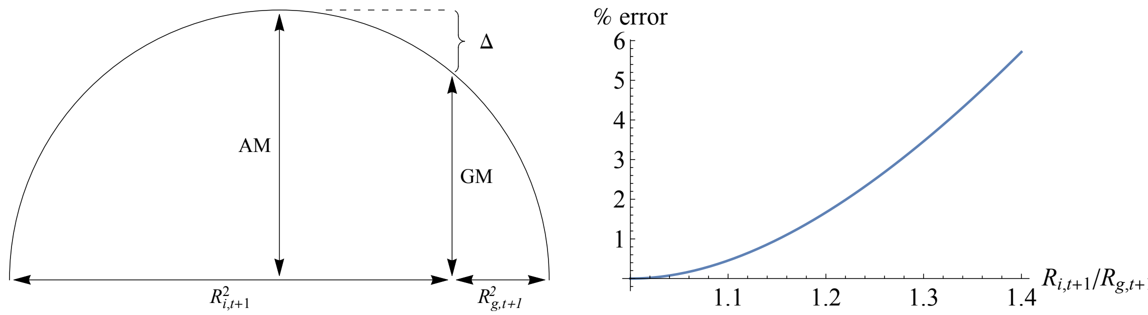
$$\mathbb{E}_t\,R_{i,t+1} = \frac{1}{R_{f,t+1}}\,\mathbb{E}_t^*\left(R_{i,t+1}R_{g,t+1}\right). \tag{3}$$

To make further progress, we will now exploit the fact that $R_{i,t+1}R_{g,t+1} = \frac{1}{2}(R_{i,t+1}^2 + R_{g,t+1}^2) - \frac{1}{2}(R_{i,t+1} - R_{g,t+1})^2$. Making this substitution in equation (3), we find that

$$\mathbb{E}_t\,R_{i,t+1} = \frac{1}{2R_{f,t+1}}\,\mathbb{E}_t^*\left[R_{i,t+1}^2 + R_{g,t+1}^2 - \left(R_{i,t+1} - R_{g,t+1}\right)^2\right]. \tag{4}$$

The last term inside the expectation is something of an irritant for us: we will shortly make an assumption that it takes a certain convenient form. In anticipation of that assumption, notice that if we dropped this third term completely, we would be approximating the geometric mean of $R_{i,t+1}^2$ and $R_{g,t+1}^2$ with their arithmetic mean. We will not do so, but—with one eye on the assumption to come—note that if the gross

[4] In the Internet Appendix, we rewrite the argument without making this assumption. The results are the same but the argument is slightly less easy to read.




(a) Arithmetic and geometric means (b) Percentage error

Figure 2: Left: The error, $\Delta$, associated with approximating the geometric mean $R_{i,t+1}R_{g,t+1}$ with the arithmetic mean, $(R_{i,t+1}^2 + R_{g,t+1}^2)/2$. $R_{i,t+1}^2$ and $R_{g,t+1}^2$ lie on the diameter of a semicircle. Right: Percentage error as a function of $R_{i,t+1}/R_{g,t+1}$.

returns $R_{i,t+1}$ and $R_{g,t+1}$ are not too far apart, we would only incur a small error[5] if we ignored the correction term entirely: see Figure 2. Moreover, all this takes place inside the expectation, so actually all we would need is that $R_{i,t+1}$ and $R_{g,t+1}$ should be tolerably close together on (risk-neutral) average.

Returning to equation (4), it follows (using the fact that $\mathbb{E}_t^* R_{i,t+1} = R_{f,t+1}$ for any gross return $R_{i,t+1}$) that

$$\frac{\mathbb{E}_t R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \frac{1}{2} \operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} + \frac{1}{2} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} + \Delta_{i,t} \tag{5}$$

where we define $\Delta_{i,t} = -\frac{1}{2} \operatorname{var}_t^* \left[ \left( R_{i,t+1} - R_{g,t+1} \right) / R_{f,t+1} \right]$. We have not made any assumptions yet: equation (5) holds as an identity. The term $\Delta_{i,t}$ would drop out if we ignored the distinction between arithmetic and geometric averages. As already noted, we will not do so, but we *will* henceforth assume that it can be decomposed as $\Delta_{i,t} = \alpha_i + \lambda_t$. (We could have considered other simplifying assumptions on the structure of $\Delta_{i,t}$—perhaps that $\Delta_{i,t} = \alpha_i \lambda_t$, for example—but given that we have an *a priori* reason to expect that $\Delta_{i,t}$ is small, we prefer the econometrically more convenient additive formulation.) We can then without further loss of generality assume that $\sum_i w_{i,t} \alpha_i = 0$, where $w_{i,t}$ is the weight of stock $i$ in the stock market index.

[5]E.g., if the net returns are 20% and 5% then $R_{i,t+1}R_{g,t+1} = 1.260$ and $(R_{i,t+1}^2 + R_{g,t+1}^2)/2 = 1.271$.

Multiplying equation (5) by $w_{i,t}$ and summing over $i$, we find that

$$\frac{\mathbb{E}_t\, R_{m,t+1} - R_{f,t+1}}{R_{f,t+1}} = \frac{1}{2}\sum_i w_{i,t}\,\text{var}_t^*\,\frac{R_{i,t+1}}{R_{f,t+1}} + \frac{1}{2}\,\text{var}_t^*\,\frac{R_{g,t+1}}{R_{f,t+1}} + \lambda_t. \tag{6}$$

where $R_{m,t+1} = \sum_i w_{i,t} R_{i,t+1}$ is the return on the market. Subtracting equation (6) from equation (5) and rearranging, we find that

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \frac{\mathbb{E}_t\, R_{m,t+1} - R_{f,t+1}}{R_{f,t+1}} + \frac{1}{2}\left(\text{var}_t^*\,\frac{R_{i,t+1}}{R_{f,t+1}} - \sum_i w_{i,t}\,\text{var}_t^*\,\frac{R_{i,t+1}}{R_{f,t+1}}\right), \tag{7}$$

where $\sum_i w_{i,t}\alpha_i = 0$.

It will be convenient to define three different measures of risk-neutral variance:

$$\begin{aligned} \text{SVIX}_t^2 &= \text{var}_t^*\left(R_{m,t+1}/R_{f,t+1}\right) \\ \text{SVIX}_{i,t}^2 &= \text{var}_t^*\left(R_{i,t+1}/R_{f,t+1}\right) \\ \overline{\text{SVIX}}_t^2 &= \sum_i w_{i,t}\,\text{SVIX}_{i,t}^2\,. \end{aligned} \tag{8}$$

These measures can be computed directly from option prices, as we show in the next section. The $\text{SVIX}_t$ index was introduced by Martin (2016)—the name echoes the related VIX index—but the definitions of stock-level $\text{SVIX}_{i,t}$ and of $\overline{\text{SVIX}}_t$, which measures average stock volatility, are new to this paper.

Introducing these definitions into (7) we arrive at our first, purely relative, prediction about the cross-section of expected returns *in excess of the market*:

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) \quad \text{where} \quad \sum_i w_{i,t}\alpha_i = 0. \tag{9}$$

We test this prediction by running a panel regression of realized returns-in-excess-of-the-market of individual stocks $i$ onto stock fixed effects and excess stock variance $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$.

In order to answer the question posed in the title of the paper, we need to take a view on the expected return on the market itself. To do so, we exploit a result of Martin (2016), who argues that the SVIX index can be used as a forecast of the equity premium: specifically, that $\mathbb{E}_t\, R_{m,t+1} - R_{f,t+1} = R_{f,t+1}\,\text{SVIX}_t^2$. Substituting this into equation (7), we have

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \text{SVIX}_t^2 + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) \quad \text{where} \quad \sum_i w_{i,t}\alpha_i = 0. \tag{10}$$

We test (10) by running a panel regression of realized excess returns on individual stocks $i$ onto stock fixed effects, risk-neutral variance $\text{SVIX}_t^2$, and excess stock variance $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$.

We also consider, and test, the—on the face of it, somewhat optimistic—possibility that the fixed effects $\alpha_i$ that appear in the above equations are constant across stocks (and hence equal to zero, since $\sum_i w_{i,t}\alpha_i = 0$). Making this assumption in (9), for example, we have[6]

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right). \tag{11}$$

Correspondingly, if we assume that the fixed effects are constant across $i$ in (10), we end up with a formula for the expected return on a stock that has no free parameters:

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \text{SVIX}_t^2 + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right). \tag{12}$$

In Section 5, we exploit the fact that (11) and (12) require no parameter estimation—only observation of contemporaneous prices—to conduct an out-of-sample analysis, and show that the formulas outperform a range of plausible competitors.

Before we turn to the data, it is worth pausing to reassess our key assumption $\Delta_{i,t} = \alpha_i + \lambda_t$. We could state explicit conditions under which this assumption holds,[7] but a more general, more plausible, justification for the assumption lies in the observation that $\Delta_{i,t}$ itself should be expected to be small for assets $i$ whose returns are, on average, not too far from the growth-optimal return. Since one can *always* decompose $\Delta_{i,t} =$

[6]At first sight, (11) appears to lead to an inconsistency: if we "set $i = m$," it seems to imply that $\text{SVIX}_t^2 = \overline{\text{SVIX}}_t^2$, which is not true (as we discuss in Section 2 below). But "setting $i = m$" is not a legitimate move here because the value-weighted sum of $\text{SVIX}_{i,t}^2$ is not $\text{SVIX}_t^2$ but $\overline{\text{SVIX}}_t^2$. By contrast, in the familiar linear factor model approach in which risk premia are expressed in terms of covariances of returns with factors, it *is* legitimate to "set $i = m$."

[7]For example, in a 'growth-optimal market model' in which stock returns take the form $R_{i,t+1} = R_{g,t+1} + u_{i,t+1}$, this condition would amount to an assumption about the risk-neutral variances of the residuals $u_{i,t+1}$.

$\alpha_i + \lambda_t + \xi_{i,t}$ where $\sum_i w_{i,t}\alpha_i = \sum_i w_{i,t}\xi_{i,t} = 0$, our assumption is equivalent to assuming that $\xi_{i,t}$ can be neglected (or, more precisely, absorbed into the error term in the panel regressions).

All that said, we emphasize that the assumption is not appropriate for all assets. Suppose that asset $j$ is genuinely idiosyncratic—and hence has zero risk premium—but has extremely high, and perhaps wildly time-varying, variance $\text{SVIX}_{j,t}^2$. Then equation (10) cannot possibly hold for asset $j$. Our identifying assumption reflects a judgment that such cases are not relevant within the universe of stocks that we study (namely, members of the S&P 100 or S&P 500 indices) whose returns plausibly have a strong systematic component.[8] This is an empirically testable judgment, and we put it to the test below.

## 2 Three measures of risk-neutral variance

Since the three measures in equation (8) are based on risk-neutral variances, they can be constructed directly from the prices of options on the market and options on individual stocks. We start with daily data from OptionMetrics for equity index options on the S&P 100 and on the S&P 500, providing us with time series of implied volatility surfaces from January 1996 to October 2014. We obtain daily equity index price and return data from CRSP and information on the index constituents from Compustat. We also obtain data on the firms' number of shares outstanding and their book equity to compute their market capitalizations and book-to-market ratios. Using the lists of index constituents, we search the OptionMetrics database for all firms that were included in the S&P 100 or S&P 500 during our sample period, and obtain volatility surface data for these individual firms, where available. Using this data, we compute the three measures of risk-neutral variance given in equation (8) for horizons (i.e., option maturities) of one, three, six, and twelve months.

As summarized in Panel A of Table 1, we end up with more than two million firm-day observations for each of the four horizons, covering a total of 869 firms over our

[8] There is a close parallel with an earlier debate on the testability of the arbitrage pricing theory (APT). Shanken (1982) showed, under the premise of the APT that asset returns are generated by a linear factor model, that it is possible to construct portfolios that violate the APT prediction that assets' expected returns are linear in the factor loadings. Dybvig and Ross (1985) endorsed the mathematical content of Shanken's results but disputed their interpretation, arguing that the APT can be applied to certain types of asset (for example, stocks), but not to arbitrary portfolios of assets.

sample period from January 1996 to October 2014. Across horizons, we have data on 451 to 453 firms on average per day, meaning that we cover slightly more than 90% of the firms included in the S&P 500 index. From the daily data, we also compile data subsets at a monthly frequency for firms included in the S&P 100 (Panel B) and the S&P 500 (Panel C).

We first show how to calculate the risk-neutral variance of the market. Martin (2016) shows that the SVIX index can be computed from the following formula:

$$\mathrm{SVIX}_t^2 = \frac{2}{R_{f,t+1} S_{m,t}^2} \left[ \int_0^{F_{m,t}} \mathrm{put}_{m,t}(K)\, dK + \int_{F_{m,t}}^{\infty} \mathrm{call}_{m,t}(K)\, dK \right].$$

We write $S_{m,t}$ and $F_{m,t}$ for the spot and forward (to time $t+1$) prices of the market, and $\mathrm{put}_{m,t}(K)$ and $\mathrm{call}_{m,t}(K)$ for the time-$t$ prices of European puts and calls on the market, expiring at time $t+1$ with strike $K$. (The length of the period from time $t$ to time $t+1$ should be understood to vary according to the horizon of interest. Thus we will be forecasting 1-month returns using the prices of 1-month options, 3-month returns using the prices of 3-month options, and so on.) The SVIX index (squared) therefore represents the price of a portfolio of out-of-the-money puts and calls equally weighted by strike. This definition is closely related to that of the VIX index, the key difference being that VIX weights option prices in inverse-square proportion to their strike. The analogous index at the individual stock level is $\mathrm{SVIX}_{i,t}$:

$$\mathrm{SVIX}_{i,t}^2 = \frac{2}{R_{f,t+1} S_{i,t}^2} \left[ \int_0^{F_{i,t}} \mathrm{put}_{i,t}(K)\, dK + \int_{F_{i,t}}^{\infty} \mathrm{call}_{i,t}(K)\, dK \right],$$

where the subscripts $i$ indicate that the reference asset is stock $i$ rather than the market.

In our empirical work, we face the issue that S&P 100 index options and individual stock options are American-style rather than European-style. Since the options whose prices we require are out-of-the-money, the distinction between the American and European options is likely to be relatively minor at the horizons we consider; in any case, the volatility surfaces reported by OptionMetrics deal with this issue via binomial tree calculations that aim to account for early exercise premia. We take the resulting volatility surfaces as our measures of European implied volatility; among others, Carr and Wu (2009) take the same approach.

Finally, using $\mathrm{SVIX}_{i,t}^2$ for all firms available at time $t$, we calculate the risk-neutral average stock variance index as $\overline{\mathrm{SVIX}}_t^2 = \sum_i w_{i,t}\, \mathrm{SVIX}_{i,t}^2$, where $w_{i,t}$ denotes the weight of firm $i$ determined by its relative market capitalization. We note that, as a matter of theory, average stock volatility *must* exceed market volatility, that is, $\overline{\mathrm{SVIX}}_t > \mathrm{SVIX}_t$. In view of the definitions above, this is an instance of the fact that a portfolio of options is more valuable than an option on a portfolio. (More formally, it is a consequence of the fact that $\sum_i w_{i,t} \operatorname{var}_t^* R_{i,t+1} > \operatorname{var}_t^* \sum_i w_{i,t} R_{i,t+1}$ or, equivalently, that $\mathbb{E}_t^* \sum_i w_{i,t} R_{i,t+1}^2 > \mathbb{E}_t^* \left[ \left( \sum_i w_{i,t} R_{i,t+1} \right)^2 \right]$, which follows from Jensen's inequality.)

Figures 3 and 4 plot the time series of risk-neutral market variance ($\mathrm{SVIX}_t^2$) and average risk-neutral stock variance ($\overline{\mathrm{SVIX}}_t^2$) for the S&P 100 and S&P 500, respectively. The dynamics of $\mathrm{SVIX}_t^2$ and $\overline{\mathrm{SVIX}}_t^2$ are similar for S&P 100 and S&P 500 stocks. All the time series spike dramatically during the financial crisis of 2008. While the average levels of the SVIX measures are similar across horizons, their volatility is higher at short than at long horizons. Similarly, the peaks in $\mathrm{SVIX}_t^2$ and $\overline{\mathrm{SVIX}}_t^2$ during the crisis and other periods of heightened volatility are most pronounced in short-maturity options.

As we show in the appendix, the ratio of market variance to average stock variance, $\mathrm{SVIX}_t^2 / \overline{\mathrm{SVIX}}_t^2$, can be interpreted as a measure of average risk-neutral correlation between stocks; see Appendix B. Figure 5 plots the time-series of $\mathrm{SVIX}_t^2 / \overline{\mathrm{SVIX}}_t^2$ at one-month and one-year horizons, for both the S&P 100 and S&P 500. Average stock variance was unusually high relative to market variance over the period from 2000 to 2002, indicating that the correlation between stocks was unusually low at that time.

To take a first look at the relation between risk-neutral stock variances and firm characteristics, we sort stocks into portfolios based on their CAPM beta, size, book-to-market ratio, or momentum, and compute the (equally-weighted) average $\mathrm{SVIX}_{i,t}^2$ for each portfolio, at the 12-month horizon.[9] $\mathrm{SVIX}_{i,t}^2$ is positively related to CAPM beta and inversely related to firm size, both on average (Figure 6, Panels A and B) and throughout our sample period (Figure 7, Panels A and B). In contrast, there is a U-shaped relationship between $\mathrm{SVIX}_{i,t}^2$ and book-to-market (Figure 6, Panel C) that reflects an interesting time-series relationship between book-to-market and $\mathrm{SVIX}_{i,t}^2$.

[9] We measure momentum by the return over the past twelve months, skipping the most recent month's return (see, e.g., Jegadeesh and Titman, 1993). Our estimation of conditional CAPM betas based on past returns follows Frazzini and Pedersen (2014), i.e. we estimate volatilities by one-year rolling standard deviations of daily returns and correlations from five-year rolling windows of overlapping three-day returns.

Growth and value stocks had similar levels of volatility during periods of low index volatility, but value stocks were more volatile than growth stocks during the recent financial crisis and less volatile from 2000 to 2002 (Figure 7, Panel C). In the case of momentum, we also find a non-monotonic relationship on average (Figure 6, Panel D); and that loser stocks exhibited particularly high $\text{SVIX}^2_{i,t}$ from late 2008 until the momentum crash in early 2009 (Figure 7, Panel D).[10]

# 3 Testing the model

In this section, we use $\text{SVIX}^2_t$, $\text{SVIX}^2_{i,t}$ and $\overline{\text{SVIX}}^2_t$ to test the predictions of our model using full sample information. But before turning to formal tests, we conduct a preliminary exploratory exercise. Specifically, we ask whether, on time-series average, stocks' average excess returns line up with their excess stock variances in the manner predicted by equation (12). To do so, we restrict to firms that were included in the S&P 500 throughout our sample period. For each such firm, we compute time-averaged excess returns and risk-neutral excess stock variance, $\text{SVIX}^2_i - \overline{\text{SVIX}}^2$. Equation (12) implies that for each percentage point difference in $\text{SVIX}^2_i - \overline{\text{SVIX}}^2$, we should see half that percentage point difference in excess returns.

The results of this exercise are shown in Figure 8, which is analogous to the security market line of the CAPM. The return horizon matches the maturity of the options used to compute the SVIX-indices. We regress average excess returns on $0.5 \times (\text{SVIX}^2_i - \overline{\text{SVIX}}^2)$; the regression intercept equals the implied average equity premium. We find slope coefficients of 0.66, 0.85, 1.03, and 1.12 for horizons of one, three, six, and twelve months, respectively—close to the model-predicted coefficient of one—and $R^2$ ranging from 0.11 to 0.20. The figures also show 3x3 portfolios sorted on size and book-to-market, indicated by triangles.

We repeat this exercise for portfolios sorted on firms' risk-neutral variance $\text{SVIX}_i$, CAPM beta, size, book-to-market, and momentum,[11] using all available firms (lifting the requirement of full sample period coverage). Figures 9 and 10 show that average

[10] We find similar results at the 1-month horizon: see Figures IA.1 and IA.2 in the Internet Appendix. Figure IA.3 plots (equally-weighted) average $\text{SVIX}_i$ at the 12-month horizon for portfolios double-sorted on size and value.

[11] Figure IA.4, in the Internet Appendix, shows results for portfolios double-sorted on size and book-to-market.

portfolio returns in excess of the market are broadly increasing in portfolios' average volatility relative to aggregate stock volatility, and that $\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$ captures a sizeable fraction of the cross-sectional variation in returns.

To test the model formally, we estimate the pooled regression

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\, \text{SVIX}_t^2 + \gamma \left( \text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}, \tag{13}$$

which constitutes a test of our formula (12) for the expected return on a stock; based on the theoretical results of the previous section, we would ideally hope to find that $\alpha = 0$, $\beta = 1$ and $\gamma = 1/2$. At a given point in time $t$, our sample includes all firms that are time-$t$ constituents of the index that serves as a proxy for the market. We run the regression using monthly data, for return horizons (and hence also option maturities) of one, three, six, and twelve months.

We test the prediction (10) by running a panel regression with firm fixed effects

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\, \text{SVIX}_t^2 + \gamma \left( \text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}. \tag{14}$$

In this form, the prediction of our theory is that $\beta = 1$, $\gamma = 1/2$, and $\sum_i w_{i,t}\alpha_i = 0$. We compute standard errors for both regressions via a block-bootstrap procedure that accounts for time-series and cross-sectional dependencies in the data.[12]

The pooled regression results for S&P 100 firms are shown in Panel A of Table 2. The headline result is that when we conduct a Wald test of the joint hypothesis that $\alpha = 0$, $\beta = 1$, and $\gamma = 0.5$, we do not reject our model at any horizon (with $p$-values ranging from 0.48 to 0.66). By contrast, we can reject the joint hypothesis that $\beta = 0$ and $\gamma = 0$ with moderate confidence for six- and twelve-month returns ($p$-values of 0.071 and 0.043, respectively), though not at shorter horizons. The point estimates of $\beta$ are 0.027, 1.094, 2.286, and 1.966 for horizons of one, three, six, and twelve months,

[12] Petersen (2009) provides an extensive discussion of how cross-sectional and time-series dependencies may bias standard errors in OLS regressions and suggests using two-way clustered standard errors. We take a more conservative approach for two reasons. First, our monthly data generates overlapping observations at return horizons exceeding one month. Second, our data is characterized by very high but less than perfect coverage of index constituent firms, due to limited availability of option data. We therefore use an overlapping block resampling scheme to handle serial correlation and heteroskedasticity, and for every bootstrap iteration, we randomly choose which constituent firms to include in the bootstrap sample. For more details on block bootstrap procedures see, e.g., Kuensch (1989), Hall et al. (1995), Politis and White (2004), and Patton et al. (2009).

respectively. These numbers are consistent with our model, but the coefficients are imprecisely estimated so are not (individually) significantly different from zero. The point estimates of $\gamma$ are 0.340, 0.397, 0.664, and 0.840 at the four horizons. At horizons of six and twelve months, these estimates are (individually) significantly different from zero, indicating a strong, positive relation between firms' equity returns and their excess stock variance.

Accounting for firm fixed effects (in Panel B) does not change this conclusion, and encouragingly the coefficient estimates remain reasonably stable. A Wald test of the joint null hypothesis that $\Sigma_i w_{i,t}\alpha_i = 0$, $\beta = 1$, and $\gamma = 0.5$ does not reject the model ($p$-values between 0.11 and 0.42), and we can strongly reject the joint null that $\beta = \gamma = 0$ for horizons of six and twelve months ($p$-values of 0.020 and 0.000, respectively). The $\beta$ estimates are little changed compared to the pooled regressions. The $\gamma$ estimates are somewhat higher—from 0.533 to 1.230—and more significant with bootstrap $t$-statistics of 1.65, 2.66, and 3.93 at horizons of three, six, and twelve months. We also find, in line with our theory, that the value-weighted sum of firm fixed effects, $\sum_i w_{i,t}\alpha_i$, is not statistically different from zero.[13]

We find similar results for S&P 500 firms (Table 3). In the pooled regression (Panel A), we do not reject the joint null that $\alpha = 0$, $\beta = 1$, and $\gamma = 0.5$ at any horizon ($p$-values between 0.154 and 0.185); and we can cautiously reject the joint null that $\beta = 0$ and $\gamma = 0$ at horizons of six and twelve months ($p$-values of 0.072 and 0.092). The statistical results are more clear-cut when we take firm fixed effects into account, in Panel B. Again, we do not reject the joint null hypothesis implied by our model at any horizon, but can strongly reject the null that $\beta = \gamma = 0$ at horizons of six and twelve months ($p$-values of 0.023 and 0.008).

To focus on the purely cross-sectional implications of our framework, we test the prediction of equation (11) by running the regression

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left( \text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}, \tag{15}$$

[13] The estimate reported for $\Sigma_i w_{i,t}\alpha_i$ is the time-series average of the value-weighted sum of firm fixed effects computed every period in our sample. The $t$-statistic is based on the distribution of the time-series average of $\Sigma_i w_{i,t}\alpha_i$ across bootstrap iterations.

and (corresponding to equation (9)) a regression that allows for stock fixed effects,

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}. \tag{16}$$

The theoretical prediction is then that $\alpha = 0$ and $\gamma = 1/2$ in equation (15), and that $\sum_i w_{i,t}\alpha_i = 0$ and $\gamma = 1/2$ in equation (16). These regressions avoid any reliance on the argument of Martin (2016) that the equity premium can be proxied by $\mathrm{SVIX}_t^2$, though of course they only make a relative statement about the cross-section of expected returns. In our empirical analysis we compute $R_{m,t+1}$ as the return on the value-weighted portfolio of all index constituent firms included in our sample at time $t$.

Tables 4 and 5 report the regression results for S&P 100 and S&P 500 firms. The results are consistent with the preceding evidence. For the pooled regressions, the estimated intercepts $\alpha$ are statistically insignificant, while the estimates of $\gamma$ are significant at horizons of six and twelve months. The Wald tests of the joint hypothesis that $\alpha = 0$ and $\gamma = 0.5$ do not reject our model ($p$-values between 0.368 and 0.801). Accounting for firm fixed effects, the significance of the $\gamma$-estimate becomes more pronounced across return horizons, with $p$-values of the null hypothesis $\gamma = 0$ being 0.10 or less even at the shorter horizons. We also find that the value-weighted sum of firm fixed effects is statistically different from zero, though the estimates are fairly small in economic terms—and, reassuringly, we will see in the next section that our model performs well when we drop firm fixed effects entirely, as we do in our out-of-sample analysis.

It is natural to worry that these positive results may be overturned by sorting stocks on characteristics known to prove troublesome for previous generations of models. To address this concern, we explore whether our theory can successfully forecast the returns of portfolios sorted by CAPM beta, size, book-to-market, and momentum. At the end of each month, we sort S&P 100 (S&P 500) firms into decile (25) portfolios and compute the portfolios' (equally-weighted) returns in excess of the market as well as the portfolios' stock variance in excess of aggregate stock variance. Using these portfolio series, we run the regressions specified in equations (15) and (16). (We also run the corresponding regressions (13) and (14) for conventional excess returns; see Tables IA.1 and IA.2 of the Internet Appendix. Since the conclusions are similar, we have chosen to focus our discussion on returns in excess of the market in order to emphasize the purely cross-sectional dimension of the data.)

The results for S&P 100 and S&P 500 stock portfolios are shown in Tables 6 and 7, respectively. The model performs well for portfolios sorted on beta, book-to-market, and momentum: we do not reject the model for any of the three characteristics for either the S&P 100 or S&P 500. We are able to reject the model for size-sorted portfolios, however: we find that estimates of $\gamma$ significantly exceed the theoretical value of 0.5, so that stock-level risk-neutral volatility forecasts returns even more strongly than our results predict. In Tables IA.3 and IA.4 of the Internet Appendix, we report results for portfolios of S&P 500 stocks formed by conditional double sorts on characteristics and $\text{SVIX}^2_{i,t}$. These results strengthen the evidence presented in this section: we do not reject our model for any of the characteristics—including size—on the double-sorted portfolios.

# 4 (Un)expected returns and firm characteristics

The results above show that the model performs well in forecasting stock returns. Nonetheless, we would like to know whether there is return-relevant information in other firm characteristics that is not captured by our predictor variables (see, e.g., Fama and French, 1993; Carhart, 1997; Lewellen, 2015).

We therefore explore the relationship between CAPM beta, log size, book-to-market ratio, and past return[14] and (i) realized excess returns (since this permits us to run a horse-race between our predictor variables and characteristics); (ii) expected excess returns (in order to determine what types of stocks have high risk premia, according to our framework); and, for completeness, (iii) unexpected excess returns (the difference between realized and expected excess returns). Throughout this exercise, we work with S&P 500 firms and at the annual horizon. In order to focus on the cross-sectional predictions of our model, we calculate excess returns relative to the market; the corresponding results for conventional excess returns are reported in Table IA.5 of the Internet Appendix, and are very similar.

The results of these exercises are shown in Table 8. The first two columns report the relationship between *realized* excess returns and characteristics. We do not

[14]We measure the firm's past return over the last year, skipping the most recent month's return. This definition corresponds to the one used above to generate momentum portfolios.

find a significant relationship between realized excess returns and beta, size, book-to-market, or past return, and the four characteristics together explain only about 1.0% of the variation in realized excess returns. When we include excess stock variance, $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$, as an additional explanatory variable the adjusted-$R^2$ quadruples to 4.0%; moreover, the estimated coefficient on stock variance is statistically different from zero but not from the value of 0.5 predicted by our framework. We reject, via Wald tests, the null hypothesis that the coefficients on characteristics and excess variance are jointly zero (with a $p$-value of 0.019); but we do not reject the joint hypothesis that the coefficients on the characteristics are zero and the coefficient on excess variance is 0.5 (with a $p$-value of 0.240). These results support our approach.

The next columns of the table address the relationships between *expected* excess returns and characteristics. The third column reports the theory-implied expected excess return given in equation (11); the fourth reports expected excess return using the coefficients estimated in the regression (15). The characteristics capture a sizeable fraction—about 38%—of the variation in expected returns. When we use the formula (11) to forecast stocks' returns in excess of the market, we find a significantly positive relationship between expected excess returns and beta; a significantly negative relationship between expected excess returns and log size; and statistically insignificant relationships between expected excess returns and book-to-market and past return. When we use the coefficient estimates from regression (15) rather than the theoretical parameter values as in equation (11), the estimates of the regression coefficients for the characteristics are very similar but they are neither individually nor jointly significant.

The last two columns of the table show that there is little evidence of a systematic relationship between *unexpected* returns and characteristics. The coefficient on beta is individually significant in one of the two specifications, but we do not reject the joint hypothesis that all coefficients are zero (with $p$-values of 0.161 and 0.623, depending on which definition of expected return is used).

# 5 Out-of-sample analysis

The formulas (11) and (12) have no free parameters; and they emerged from a small amount of theory (as opposed to a large amount of data-mining). It is therefore reasonable to hope that they may be well suited to out-of-sample forecasting.[15]

In this section, we show that they are. As will become clear, the formulas perform well out-of-sample, especially over the longer return horizons. This fact is particularly striking because—as shown in Figure 1 in the cases of Apple and Citibank—they make aggressive predictions both in the time series (there are some points in time when forecast returns are extremely high on average) and in the cross-section (at a fixed point in time, the formulas imply substantial cross-sectional variation in expected returns). The former point is consistent with Martin (2016); the latter is new to this paper. It is illustrated in Figure 11, which plots the evolution of the cross-sectional standard deviation of the expected return forecasts generated by the formula (12) over time. The cross-sectional standard deviation averages 4.4% over our sample period and peaks above 15% during the subprime crisis. These are large numbers. For comparison, the figure also shows that the corresponding cross-sectional standard deviation of expected return forecasts based on the CAPM is around a third as large on average, and only about a tenth as volatile in the time series.

## 5.1 Statistical forecast accuracy

We compare the performance of the formulas (11) and (12) to various competitor forecasting benchmarks using an out-of-sample R-squared measure along the lines of Goyal and Welch (2008). We define

$$R^2_{OS} = 1 - \frac{\sum_i \sum_t FE^2_{M,it}}{\sum_i \sum_t FE^2_{B,it}},$$

where $FE_{M,it}$ and $FE_{B,it}$ denote the forecast errors for stock $i$ at time $t$ based on our model and on a benchmark prediction, respectively. Our model outperforms a given benchmark if the corresponding $R^2_{OS}$ is positive.

### 5.1.1 Out-of-sample benchmark forecasts

What are the natural competitor benchmarks? One possibility is to give up on trying to make differential predictions across stocks, and simply to use a forecast of the expected

[15] By this we simply mean that the forecasts made by the two formulas do not rely on in-sample information, for the strong reason that the formulas do not depend on any parameters at all.

return on the market as a forecast for each individual stock. We consider various ways of doing so. We use the market's historical average excess return as an equity premium forecast, following Goyal and Welch (2008) and Campbell and Thompson (2008), and we use the S&P 500 ($\overline{\text{S\&P500}}_t$) and the CRSP value-weighted index ($\overline{\text{CRSP}}_t$) as proxies for the market. We also use the risk-neutral variance of the market, $\text{SVIX}_t^2$, to proxy for the equity premium, as advocated by Martin (2016). Lastly, we consider a constant excess return forecast of 6% *p.a.*, corresponding to long-run estimates of the equity premium used in previous research.

More ambitious competitor models would seek to provide differential forecasts of individual firm stock returns, as we do. Again, we consider several alternatives. One natural thought is to use historical average of firms' stock excess returns ($\overline{\text{RX}}_{i,t}$). Another is to estimate firms' conditional CAPM betas from historical return data and combine the beta estimates with the aforementioned market premium predictions. Yet another is motivated by Kadan and Tang (2016), who show that under certain conditions $\text{SVIX}_{i,t}^2$ provides a lower bound on stock $i$'s risk premium: we therefore also use firms' risk-neutral variances ($\text{SVIX}_{i,t}^2$) as a competitor forecasting variable.

The results for expected excess returns are shown in Panel A of Table 9. Our formula (12) outperforms all the above competitors at the 3-, 6-, and 12-month horizons, and its relative performance (as measured by $R_{OS}^2$) almost invariably increases with forecast horizon.[16] At the one-year horizon, $R_{OS}^2$ ranges from 1.7% to 3.8% depending on the competitor benchmark, with the exception of the historical average stock return $\overline{\text{RX}}_{i,t}$, which it outperforms by a wide margin, with an $R_{OS}^2$ above 27%.[17]

The results for expected returns in excess of the market are shown in Panel B and are, if anything, even stronger. We adjust the conditional CAPM predictions appropriately (by multiplying the equity premium by beta minus 1); and we add a 'random walk' forecast of zero. In doing so, we focus on the cross-sectional dimension of firms' equity returns, net of (noisy) market return forecasts. The formula (11) outperforms all the competitors at every horizon, and the outperformance increases

[16] The $R_{OS}^2$ results are based on expected excess returns defined as $\mathbb{E}_t\, R_{i,t+1} - R_{f,t+1}$, i.e. we multiply the left and the right side of equations (11) and (12) by $R_{f,t+1}$.

[17] It is a strength of our approach that it does not rely on historical data: for individual firms, short historical return series may not be representative of future returns. Consider, for instance, tech firms that were relatively young at the peak of the dotcom bubble, and therefore had extremely high historical average returns over their short histories. In such cases, employing the historical average as a predictor may lead to large forecast errors for subsequent returns.

with forecast horizon. At the one-year horizon, $R^2_{OS}$ is around 3% relative to each of the benchmarks.

### 5.1.2 In-sample benchmark predictions

Even more ambitiously, we can compare the performance of our model's out-of-sample forecasts to benchmark predictions that exploit *in-sample* information about the average excess return on the market, the average excess return across all stocks, and the in-sample predictive relationships between excess returns and firm characteristics. We report results for excess returns and returns in excess of the market in Panels A and B of Table 10, respectively.

We start with excess returns in Panel A. The first three lines of the table show the formula's performance relative to the in-sample average equity premium and the in-sample average excess return on a stock (each of which makes the same forecast for every stock's return); and to estimated beta multiplied by the in-sample equity premium (which does differentiate across stocks). In each case, $R^2_{OS}$ is increasing with forecast horizon and is positive at horizons of three, six, and twelve months.

Next, we evaluate our model forecasts relative to in-sample predictions based on firm characteristics—that is, based on the fitted values from pooled univariate regressions of excess returns onto conditional betas, onto (log) size, onto book-to-market ratios, or onto the stock's past return. Our formula outperforms at horizons of six and twelve months. The only case in which we 'lose' is when we compare to the best in-sample multivariate prediction based on all four characteristics, in which case $R^2_{OS}$ ranges from $-0.53\%$ to 0.20%.

Our model performs even better when forecasting returns in excess of the market. At horizons of three, six and 12 months, the formula beats all the competitors, *even the in-sample regression prediction based on all four characteristics simultaneously.* Its predictive performance is best (relative to the competitors) at longer horizons: at the 12-month horizon, $R^2_{OS}$ ranges from 1.5% to 3.3%.

## 5.2 Economic value of model forecasts

To assess whether conditioning on stock return forecasts from our model generates substantial economic value for investors, we now compare the performance of a portfolio with weights determined by model forecasts to a value-weighted portfolio and to an equally-weighted portfolio. The value-weighted portfolio serves as a natural benchmark because it mimics the market.[18] The equally-weighted portfolio has proven in previous research to be a tough benchmark to beat (see, e.g. DeMiguel et al., 2009) and therefore serves as a natural competitor as well. Using both the equally- and the value-weighted portfolios as benchmarks takes size effects into account to some extent, but we also investigate how the performance of our portfolios relates to firm size and book-to-market in more detail below.

We have shown that our framework successfully forecasts expected excess returns out of sample. It therefore supplies one of the two key ingredients for mean-variance analysis—indeed, it supplies the ingredient that is traditionally viewed as harder to estimate. We are currently working on combining the risk premium forecasts that emerge from option prices, as here, with estimates of the covariance matrix based on historical price data, but this exercise is beyond the scope of the present paper. For now, we therefore explore a trading strategy that only requires expected returns as a signal for investment decisions.

More specifically, we draw on the idea of Asness et al. (2013) and construct portfolios based on the ranks of firms' expected returns, where we determine the portfolio weight of firm $i$ for the period from $t$ to $t+1$ as

$$w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t\, R_{i,t+1}]^{\theta}}{\sum_i \text{rank}[\mathbb{E}_t\, R_{i,t+1}]^{\theta}}. \tag{17}$$

The parameter $\theta \geq 0$ controls the aggressiveness of the strategy: as $\theta$ increases, the relative overweighting of firms with high expected returns becomes more pronounced but the tilting from equal weights remains moderate for reasonable values of $\theta$.[19] This methodology assigns weights that increase with expected excess returns; it does not

[18]More precisely, the value-weighted portfolio mimics the market to the extent that constituent firms are included in the sample at time $t$, which is subject to availability of equity options data. In our analysis of out-of-sample economic value, we cover more than 90% of S&P 500 firms on average. Over our full sample period the return on the value-weighted portfolio is somewhat higher than that of the S&P 500 index, but the monthly excess return correlation exceeds 0.999.

[19]For $\theta = 0$, the strategy corresponds to the equally-weighted portfolio benchmark. In our empirical analysis, we set $\theta = 1$ or $\theta = 2$ which leads to over- or underweighting of stocks relative to the equal-weighted benchmark but avoids extreme positions and ensures that the portfolio is well diversified. For 500 stocks, an equal-weighted strategy allocates a 0.2%-weight to each asset in the portfolio; the maximum rank weights assigned to a single stock is 0.399% for $\theta = 1$ and 0.598% when setting $\theta = 2$.

allow for short positions and (by avoiding extreme positions) ensures that the portfolio is well-diversified.

To evaluate the portfolio relative to the benchmarks, we compute returns, sample moments, Sharpe ratios, and the performance fee as defined by Fleming et al. (2001). For a given relative risk aversion $\rho$, the performance fee ($\Phi$) quantifies the premium that a risk-averse investor would be willing to pay to switch from the benchmark investment ($B$) to the portfolio conditioning on the forecasts from our model ($M$). Specifically, $\Phi$ is computed as a fraction of initial wealth ($W_0$) such that two competing portfolio strategies achieve equal average utility. Denoting the gross portfolio return of portfolio $k \in \{B, M\}$ by $R_{k,t+1}$, we compute the average realized utility, $\overline{U}\left(\{R_{k,t+1}\}\right)$, as

$$\overline{U}\left(\{R_{k,t+1}\}\right) = W_0 \left( \sum_{t=0}^{T-1} R_{k,t+1} - \frac{\rho}{2(1+\rho)} R_{k,t+1}^2 \right)$$

and estimate the performance fee as the value $\Phi$ that satisfies

$$\overline{U}\left(\{R_{M,t+1} - \Phi\}\right) = \overline{U}\left(\{R_{B,t+1}\}\right).$$

We start by considering an investor who rebalances her portfolio at the end of every month. She may either invest in the market by choosing the value-weighted portfolio or the equally-weighted portfolio, or use the model forecasts to determine the portfolio weights according to equation (17) where we set $\theta = 2$.[20] Table 11 compares the performance of these three alternatives. Different columns now represent the horizon of SVIX used to forecast returns; independent of this horizon, we update the portfolio weights monthly. The forecast-based portfolios generate annualized excess returns between 10.85% and 12.53% with associated Sharpe ratios between 0.44 and 0.51, where excess returns and Sharpe ratios moderately increase with the SVIX horizon used.

The model-based portfolios dominate the value-weighted portfolios in terms of Sharpe ratios and skewness properties, and as a result investors would be willing to pay a sizeable fee to switch from the benchmark to the model portfolios. The estimated fee, $\Phi$, depends on the investor's relative risk aversion: setting $\rho$ between one and ten, we find that the annual performance fee is in the range of 2.32% to 5.20%. When

[20]Tables IA.6 and IA.7 in the Internet Appendix report the corresponding results for monthly and daily rebalanced portfolios with $\theta = 1$.

we compare the model portfolio to the equal-weighted portfolio, the estimates of $\Phi$ are again positive, albeit lower. The equally-weighted portfolios have slightly higher Sharpe ratios than the model portfolios but less favorable skewness properties, so that investors would be willing to pay performance fees between 0.05% to 2.13% per year to switch to the model portfolio.

A major advantage of our approach is that it allows the investor to update her expectations about each firm in real time. Table 12 illustrates this advantage by showing how performance improves when the investor rebalances her portfolios at the end of every day, rather than every month. Daily rebalancing widens the gap between model and value-weighted portfolio Sharpe ratios, while the Sharpe ratios of the model portfolios are similar to those of the equal-weighted portfolios. The model portfolio has less negative skewness than either the equal-weighted or value-weighted portfolios. The estimates of $\Phi$ increase compared to the monthly rebalancing results, with investors willing to pay performance fees in the range between 4.14% to 5.41% compared to the value-weighted portfolios and 1.23% to 2.42% compared to the equally-weighted portfolios.

Figure 12 illustrates these results by plotting the cumulative returns on the model portfolio (using forecasts based on SVIX horizons of one month and one year in Panels A and B, respectively); on the value- and equal-weighted index; and on cash.

We repeat the above analysis in subsets of S&P 500 stocks based on firm size and book-to-market. Table 13 shows that the model generates higher utility gains for small firms than for big firms, and for value stocks than for growth stocks. Accordingly, we find the highest utility gains compared to the benchmarks for small value firms (in the range from 7% to 11%) and the smallest fees for big growth stocks (between 1.4% and 3%). Panels C to F in Figure 12 plot the cumulative returns on the model portfolio and benchmark portfolios in each of the four size/value categories.

# 6 Conclusions

This paper has presented new theoretical and empirical results on the cross-section of expected stock returns. We would like to think that our approach to this classic topic is idiosyncratic in more than one sense.

Our empirical work is tightly constrained by our theory. In sharp contrast with the factor model approach to the cross-section—which has both the advantage and the disadvantage of imposing very little structure, and therefore says *ex ante* little about the anticipated signs, and nothing about the sizes, of coefficient estimates—we make specific predictions both for the signs and sizes of coefficients, and test these numerical predictions in the data. In this dimension, a better comparison is with the CAPM, which makes the quantitative prediction that the slope of the security market line should equal the market risk premium. But (setting aside the fact that it makes no prediction for the market risk premium) even the CAPM requires betas to be estimated if this prediction is to be tested. At times when markets are turbulent, it is far from clear that historical betas provide robust measures of the idealized forward-looking betas called for by the theory; and if the goal is to forecast returns over, say, a one-year horizon, one cannot respond to this critique by taking refuge in the last five minutes of high-frequency data. In contrast, our predictive variables—as option prices—are observable in real time and inherently forward-looking.

The empirical evidence is surprisingly supportive of our approach, particularly over six- and twelve-month horizons. We test the model with and without fixed effects; we consider both returns in excess of the riskless rate (to answer the question posed in the title) and returns in excess of the market (to isolate the cross-sectional predictions and differentiate our findings from those of Martin (2016)); and we present results for S&P 100 and for S&P 500 stocks. At six- and twelve-month horizons we can reject the null of no predictability with some confidence, whereas we do not reject our model—and find reasonably stable coefficient estimates—in most of the specifications.

We try to find evidence *against* our model by forming portfolios sorted on four dimensions known to be problematic for previous generations of asset-pricing models. The model does a good job of accounting for realized returns on portfolios sorted on beta, book-to-market, and past returns. When we sort on size, however, we find that the sensitivity of portfolio returns to stock volatility has the 'right' sign but is even stronger than predicted by our framework, so that we are able to reject the model.

One of the strengths of the framework is that the coefficients in our formula for the expected return on a stock are theoretically motivated. To implement the formula, we need only observe the market prices of certain options: no estimation is required. Our approach therefore avoids the critique of Goyal and Welch (2008). We compare the out-of-sample performance of the formula to a range of competitor predictors of stock returns, and show that it outperforms them all. Finally, we show how our forecasts can be used in trading strategies, and show that the resulting strategies perform well.

Our results imply that expected returns are extremely volatile over time (consistent with Martin, 2016) and across stocks. As an example, expected returns on some major financial stocks were astonishingly high in the depths of the subprime crisis: on our view, the annual expected return of Citigroup rose above 100% in early 2009. More generally, the cross-sectional standard deviation of expected returns implied by the formula (12) is around three times as large, on average, as that of the corresponding forecasts based on the CAPM; and the time-series *volatility* of the cross-sectional standard deviation is between 6 and 15 times larger than for the CAPM. This evidence points toward a quantitatively—indeed qualitatively—new view of risk premia.

# Appendix

## A A proof of the claim in footnote 4

This section shows that the assumption that $R_{i,t+1}R_{g,t+1}$ is a tradable payoff is innocuous, given our maintained assumption that $\Delta_{i,t} = \alpha_i + \lambda_t$. Let $\mathbb{E}_t^*$ be a risk-neutral measure; there may be more than one, if markets are incomplete. Since $R_{i,t+1}R_{g,t+1} = \frac{1}{2}R_{i,t+1}^2 + \frac{1}{2}R_{g,t+1}^2 - \frac{1}{2}\left(R_{i,t+1} - R_{g,t+1}\right)^2$,

$$\frac{1}{R_{f,t+1}^2}\,\mathbb{E}_t^*\,R_{i,t+1}R_{g,t+1} = \frac{1}{2R_{f,t+1}^2}\left[\mathbb{E}_t^*\,R_{i,t+1}^2 + \mathbb{E}_t^*\,R_{g,t+1}^2\right] - \frac{1}{2}\operatorname{var}_t^*\frac{R_{i,t+1} - R_{g,t+1}}{R_{f,t+1}}.$$

As in the body of the paper, we assume that the final term takes the form $\alpha_i + \lambda_t$. Without further loss of generality, we may assume that $\sum_i w_{i,t}\alpha_i = 0$, so the above equation can be rewritten as

$$\frac{1}{R_{f,t+1}^2}\,\mathbb{E}_t^*\,R_{i,t+1}R_{g,t+1} - 1 = \frac{1}{2R_{f,t+1}^2}\left[\operatorname{var}_t^*\,R_{i,t+1} + \operatorname{var}_t^*\,R_{g,t+1}\right] + \alpha_i + \lambda_t. \tag{A.1}$$

Multiplying (A.1) by $w_{i,t}$ and summing over $i$, it follows that

$$\frac{1}{R_{f,t+1}^2}\,\mathbb{E}_t^*\,R_{m,t+1}R_{g,t+1} - 1 = \frac{1}{2R_{f,t+1}^2}\left[\sum_i w_{i,t}\operatorname{var}_t^*\,R_{i,t+1} + \operatorname{var}_t^*\,R_{g,t+1}\right] + \lambda_t. \tag{A.2}$$

Subtracting (A.2) from (A.1), we find that

$$\frac{1}{R_{f,t+1}^2}\,\mathbb{E}_t^*\left[\left(R_{i,t+1} - R_{m,t+1}\right)R_{g,t+1}\right] = \frac{1}{2}\left[\operatorname{var}_t^*\frac{R_{i,t+1}}{R_{f,t+1}} - \sum_i w_{i,t}\operatorname{var}_t^*\frac{R_{i,t+1}}{R_{f,t+1}}\right] + \alpha_i.$$

The terms inside the square brackets on the right-hand side are uniquely determined by option prices. So the left-hand side is too (since $\alpha_i$ is a constant).

If we set $\widehat{\mathbb{E}}_t(X) = R_{f,t+1}\,\mathbb{E}_t\,\frac{X}{R_{g,t+1}}$, then $\widehat{\mathbb{E}}_t$ is a risk-neutral measure. Hence

$$\frac{1}{R_{f,t+1}^2}\widehat{\mathbb{E}}_t\left[\left(R_{i,t+1} - R_{m,t+1}\right)R_{g,t+1}\right] = \frac{1}{2}\left[\operatorname{var}_t^*\frac{R_{i,t+1}}{R_{f,t+1}} - \sum_i w_{i,t}\operatorname{var}_t^*\frac{R_{i,t+1}}{R_{f,t+1}}\right] + \alpha_i.$$

But then by the definition of $\widehat{\mathbb{E}}$, we have our result,

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left[\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} - \sum_i w_{i,t} \operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}}\right] + \alpha_i.$$

# B A measure of correlation

This section shows that the ratio $\mathrm{SVIX}_t^2 / \overline{\mathrm{SVIX}}_t^2$ can be interpreted as an approximate measure of average risk-neutral correlation between stocks. Note first that

$$\operatorname{var}_t^* R_{m,t+1} - \sum_i w_i^2 \operatorname{var}_t^* R_{i,t+1} = \sum_{i \neq j} w_i w_j \operatorname{corr}_t^*(R_{i,t+1}, R_{j,t+1}) \sqrt{\operatorname{var}_t^* R_{i,t+1} \operatorname{var}_t^* R_{j,t+1}},$$

so we can define a measure of average correlation, $\rho_t$, as

$$\rho_t = \frac{\operatorname{var}_t^* R_{m,t+1} - \sum_i w_i^2 \operatorname{var}_t^* R_{i,t+1}}{\sum_{i \neq j} w_i w_j \sqrt{\operatorname{var}_t^* R_{i,t+1} \operatorname{var}_t^* R_{j,t+1}}}.$$

Now, we have

$$\rho_t \approx \frac{\operatorname{var}_t^* R_{m,t+1}}{\sum_i w_i^2 \operatorname{var}_t^* R_{i,t+1} + \sum_{i \neq j} w_i w_j \sqrt{\operatorname{var}_t^* R_{i,t+1} \operatorname{var}_t^* R_{j,t+1}}} = \frac{\operatorname{var}_t^* R_{m,t+1}}{\left(\sum_i w_i \sqrt{\operatorname{var}_t^* R_{i,t+1}}\right)^2}.$$

This last expression features the square of average stock *volatility*, rather than average stock variance, in the denominator. To avoid a proliferation of definitions, we approximate $\left(\sum_i w_i \sqrt{\operatorname{var}_t^* R_{i,t+1}}\right)^2 \approx \sum_i w_i \operatorname{var}_t^* R_{i,t+1}$. (The approximation neglects a Jensen's inequality term; in fact, the left-hand side is strictly smaller than the right-hand side.) This leads us to our approximate correlation measure,

$$\rho_t \approx \frac{\operatorname{var}_t^* R_{m,t+1}}{\sum_i w_i \operatorname{var}_t^* R_{i,t+1}} = \frac{\mathrm{SVIX}_t^2}{\overline{\mathrm{SVIX}}_t^2}. \quad \text{(B.1)}$$

# References

An, B.-J., Ang, A., Bali, T. G., and Cakici, N. (2014). The joint cross section of stocks and options. *Journal of Finance*, 69(5):2279–2337.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. *Journal of Finance*, 61:259–299.

Ang, A., Liu, J., and Schwarz, K. (2010). Using stocks or portfolios in tests of asset pricing models. Working paper, Columbia University.

Asness, C. S., Moskowitz, T. J., and Pedersen, L. H. (2013). Value and momentum everywhere. *Journal of Finance*, 68:929–985.

Buss, A. and Vilkov, G. (2012). Measuring equity risk with option-implied correlations. *Review of Financial Studies*, 25(10):3113–3140.

Campbell, J. Y. and Thompson, S. B. (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? *Review of Financial Studies*, 21:1509–1531.

Carhart, M. (1997). On persistence in mutual fund performance. *Journal of Finance*, 52:57–82.

Carr, P. and Wu, L. (2009). Variance risk premiums. *Review of Financial Studies*, 22(3):1311–1341.

Chang, B.-Y., Christoffersen, P., Jacobs, K., and Vainberg, G. (2012). Option-implied measures of equity risk. *Review of Finance*, 16(2):385–428.

Christoffersen, P., Fournier, M., and Jacobs, K. (2015). The factor structure in equity options. *Rotman School of Management Working Paper*.

Cochrane, J. (2005). *Asset Pricing*. Princeton University Press, revised edition.

Conrad, J., Dittmar, R., and Ghysels, E. (2013). Ex ante skewness and expected stock returns. *Journal of Finance*, 68:85–124.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *Review of Financial Studies*, 22(5):1915–1953.

Driessen, J., Maenhout, P., and Vilkov, G. (2009). The price of correlation risk: Evidence from equity options. *Journal of Finance*, 64:1377–1406.

Dybvig, P. H. and Ross, S. A. (1985). Yes, the APT is testable. *Journal of Finance*, 40(4):1173–1188.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33:3–56.

Fleming, J., Kirby, C., and Ostdiek, B. (2001). The economic value of volatility timing. *Journal of Finance*, 56(1):329–352.

Frazzini, A. and Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, 111(1):1–25.

Fu, F. (2009). Idiosyncratic risk and the cross-section of expected stock returns. *Journal of Financial Economics*, 91(1):24–37.

Goyal, A. and Welch, I. (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies*, 21:1455–1508.

Hall, P., Horowitz, J., and Jing, B. (1995). On blocking rules for the bootstrap with dependent data. *Biometrika*, 82:561–574.

Hansen, L. P. and Richard, S. F. (1987). The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. *Econometrica*, 55(3):587–613.

Herskovic, B., Kelly, B., Lustig, H., and Nieuwerburgh, S. V. (2016). The common factor in idiosyncratic volatility: Quantitative asset pricing implications. *Journal of Financial Economics*, 119(2):249–283.

Jegadeesh, N. and Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, 48(1):65–91.

Kadan, O. and Tang, X. (2016). A bound on expected stock returns. Working paper, Washington University in St. Louis.

Kuensch, H. (1989). The jacknife and the bootstrap for general stationary observations. *Annals of Statistics*, 17:1217–1241.

Lewellen, J. (2015). The cross-section of expected stock returns. *Critical Finance Review*, 4(1):1–44.

Lewellen, J. and Nagel, S. (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of financial economics*, 82(2):289–314.

Lewellen, J., Nagel, S., and Shanken, J. (2010). A skeptical appraisal of asset pricing tests. *Journal of Financial economics*, 96(2):175–194.

Long, J. B. (1990). The numeraire portfolio. *Journal of Financial Economics*, 26:29–69.

Martin, I. (2016). What is the expected return on the market? *Quarterly Journal of Economics*. Forthcoming.

Patton, A., Politis, D., and White, H. (2009). Correction to "automatic block-length selection for dependent bootstrap". *Econometric Reviews*, 28:372–375.

Petersen, M. A. (2009). Estimating standard errors in finance panel data sets: Comparing approaches. *Review of Financial Studies*, 22(1):435–480.

Politis, D. and White, H. (2004). Automatic block-length selection for dependent bootstrap. *Econometric Reviews*, 23:53–70.

Roll, R. (1973). Evidence on the "growth-optimum" model. *Journal of Finance*, 28(3):551–566.

Shanken, J. (1982). The arbitrage pricing theory: Is it testable? *Journal of Finance*, 37(5):1129–1140.

Table 1: Sample data

This Table summarizes the data used in the empirical analysis. We search the OptionMetrics database for all firms hat have been included in the S&P 100 or S&P 500 during the sample period from January 1996 to October 2014 and obtain all available volatility surface data. Panel A summarizes the number of total observations, the number of unique days and unique firms in our sample, as well as the average number of firms for which options data is available per day. For some econometric analysis, we also compile data subsets at a monthly frequency for firms included in the S&P 100 (summarized in Panel B) and the S&P 500 (Panel C).

*Panel A. Daily data*

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Observations | 2,116,480 | 2,116,073 | 2,114,678 | 2,107,484 |
| Sample days | 4,675 | 4,675 | 4,675 | 4,675 |
| Sample firms | 869 | 861 | 855 | 825 |
| Average firms/day | 453 | 453 | 452 | 451 |

*Panel B. Monthly data for S&P 100 firms*

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Observations | 21,235 | 20,847 | 20,269 | 19,121 |
| Sample months | 224 | 222 | 219 | 213 |
| Sample firms | 177 | 176 | 176 | 171 |
| Average firms/month | 95 | 94 | 93 | 90 |

*Panel C. Monthly data for S&P 500 firms*

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Observations | 102,601 | 100,593 | 97,629 | 91,815 |
| Sample months | 224 | 222 | 219 | 213 |
| Sample firms | 877 | 869 | 863 | 832 |
| Average firms/month | 458 | 453 | 446 | 431 |

## Table 2: Expected excess returns of S&P 100 firms

This Table presents results from regressing equity excess returns of S&P 100 firms on the risk-neutral variance of the market variance ($\text{SVIX}_t^2$) and the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to one year. The return horizons match the maturities of the options used to compute $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$, and $\overline{\text{SVIX}}_t^2$. Panel A reports estimates of the pooled regression specified in equation (13),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\,\text{SVIX}_t^2 + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with firm fixed effects specified in equation (14),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\,\text{SVIX}_t^2 + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_{i,t}\alpha_i$ reports the time-series average of the value-weighted sum of firm fixed effects. Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept, $\beta = 1$ and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\beta = \gamma = 0$. The row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values given in (12).

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.074 | 0.035 | -0.009 | 0.001 |
| | [1.16] | [0.47] | [-0.17] | [0.01] |
| $\beta$ | 0.027 | 1.094 | 2.286 | 1.966 |
| | [0.01] | [0.48] | [1.56] | [1.40] |
| $\gamma$ | 0.340 | 0.397 | 0.664 | 0.840 |
| | [1.07] | [1.30] | [2.05] | [2.45] |
| Pooled adj-$R^2$ (%) | 0.162 | 0.796 | 3.668 | 6.511 |
| $H_0: \alpha = 0, \beta = 1, \gamma = 0.5$ | 0.484 | 0.601 | 0.655 | 0.560 |
| $H_0: \beta = \gamma = 0$ | 0.529 | 0.428 | 0.071 | 0.043 |
| Theory adj-$R^2$ (%) | -0.053 | 0.442 | 2.461 | 3.975 |
| *Panel B. Firm fixed-effects regressions* | | | | |
| $\sum_i w_i \alpha_i$ | 0.087 | 0.050 | 0.008 | 0.018 |
| | [1.40] | [0.70] | [0.15] | [0.28] |
| $\beta$ | -0.014 | 1.020 | 2.174 | 1.802 |
| | [-0.01] | [0.44] | [1.52] | [1.36] |
| $\gamma$ | 0.533 | 0.614 | 0.977 | 1.230 |
| | [1.46] | [1.65] | [2.66] | [3.93] |
| Panel adj-$R^2$ (%) | 1.000 | 4.429 | 11.439 | 20.134 |
| $H_0: \sum_i w_i \alpha_i = 0, \beta = 1, \gamma = 0.5$ | 0.296 | 0.415 | 0.383 | 0.106 |
| $H_0: \beta = \gamma = 0$ | 0.297 | 0.251 | 0.020 | 0.000 |

## Table 3: Expected excess returns of S&P 500 firms

This Table presents results from regressing equity excess returns of S&P 500 firms on the risk-neutral variance of the market variance ($\text{SVIX}_t^2$) and the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to one year. The return horizons match the maturities of the options used to compute $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$, and $\overline{\text{SVIX}}_t^2$. Panel A reports estimates of the pooled regression specified in equation (13),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\, \text{SVIX}_t^2 + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with firm fixed effects specified in equation (14),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\, \text{SVIX}_t^2 + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_{i,t}\alpha_i$ reports the time-series average of the value-weighted sum of firm fixed effects. Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept, $\beta = 1$ and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\beta = \gamma = 0$. The row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values given in (12).

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.060 | 0.020 | -0.038 | -0.021 |
| | [0.81] | [0.26] | [-0.63] | [-0.30] |
| $\beta$ | 0.736 | 1.877 | 3.485 | 3.033 |
| | [0.32] | [0.78] | [2.22] | [1.89] |
| $\gamma$ | 0.150 | 0.279 | 0.448 | 0.513 |
| | [0.55] | [1.01] | [1.43] | [1.61] |
| Pooled adj-$R^2$ (%) | 0.066 | 0.725 | 3.176 | 4.440 |
| $H_0: \alpha = 0, \beta = 1, \gamma = 0.5$ | 0.154 | 0.185 | 0.158 | 0.182 |
| $H_0: \beta = \gamma = 0$ | 0.860 | 0.582 | 0.072 | 0.092 |
| Theory adj-$R^2$ (%) | -0.198 | 0.149 | 1.441 | 1.981 |
| *Panel B. Firm fixed-effects regressions* | | | | |
| $\sum_i w_i\alpha_i$ | 0.080 | 0.042 | -0.009 | 0.011 |
| | [1.11] | [0.55] | [-0.16] | [0.16] |
| $\beta$ | 0.634 | 1.716 | 3.189 | 2.618 |
| | [0.28] | [0.71] | [2.16] | [1.75] |
| $\gamma$ | 0.381 | 0.569 | 0.845 | 0.936 |
| | [1.29] | [1.79] | [2.54] | [3.06] |
| Panel adj-$R^2$ (%) | 0.609 | 3.957 | 10.235 | 17.157 |
| $H_0: \sum_i w_i\alpha_i = 0, \beta = 1, \gamma = 0.5$ | 0.187 | 0.210 | 0.160 | 0.131 |
| $H_0: \beta = \gamma = 0$ | 0.396 | 0.166 | 0.023 | 0.008 |

## Table 4: Expected returns in excess of the market of S&P 100 firms

This Table presents results from regressing equity returns in excess of the market on the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$ for S&P 100 firms. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to one year. The return horizons match the maturities of the options used to compute $\text{SVIX}_{i,t}^2$ and $\overline{\text{SVIX}}_t^2$. Panel A reports estimates of the pooled regression specified in equation (15),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with firm fixed effects specified in equation (16),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\gamma = 0$. The row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values given in (11).

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.010 | 0.010 | 0.007 | 0.006 |
| | [0.69] | [0.67] | [0.43] | [0.40] |
| $\gamma$ | 0.424 | 0.447 | 0.688 | 0.826 |
| | [1.35] | [1.51] | [2.14] | [2.45] |
| Pooled adj-$R^2$ (%) | 0.332 | 0.945 | 3.241 | 6.195 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.782 | 0.801 | 0.677 | 0.432 |
| $H_0 : \gamma = 0$ | 0.177 | 0.131 | 0.032 | 0.014 |
| Theory adj-$R^2$ (%) | 0.314 | 0.908 | 2.941 | 5.091 |
| *Panel B. Firm fixed-effects regressions* | | | | |
| $\sum_i w_i \alpha_i$ | 0.027 | 0.025 | 0.024 | 0.022 |
| | [2.74] | [2.94] | [2.63] | [2.51] |
| $\gamma$ | 0.598 | 0.638 | 0.968 | 1.161 |
| | [1.70] | [1.78] | [2.70] | [3.71] |
| Panel adj-$R^2$ (%) | 0.852 | 3.509 | 9.220 | 16.966 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.023 | 0.012 | 0.007 | 0.002 |
| $H_0 : \gamma = 0$ | 0.088 | 0.075 | 0.007 | 0.000 |

Table 5: Expected returns in excess of the market of S&P 500 firms

This Table presents results from regressing equity returns in excess of the market on the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right)$ for S&P 500 firms. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to one year. The return horizons match the maturities of the options used to compute $\mathrm{SVIX}^2_{i,t}$ and $\overline{\mathrm{SVIX}}^2_t$. Panel A reports estimates of the pooled regression specified in equation (15),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma\left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with firm fixed effects specified in equation (16),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma\left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\gamma = 0$. The row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values given in (11).

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.018 | 0.017 | 0.013 | 0.014 |
| | [1.24] | [1.13] | [0.82] | [0.74] |
| $\gamma$ | 0.247 | 0.383 | 0.530 | 0.554 |
| | [0.93] | [1.45] | [1.75] | [1.84] |
| Pooled adj-$R^2$ (%) | 0.098 | 0.554 | 1.678 | 2.917 |
| $H_0: \alpha = 0, \gamma = 0.5$ | 0.368 | 0.523 | 0.635 | 0.602 |
| $H_0: \gamma = 0$ | 0.353 | 0.146 | 0.080 | 0.066 |
| Theory adj-$R^2$ (%) | -0.013 | 0.460 | 1.580 | 2.692 |
| *Panel B. Firm fixed-effects regressions* | | | | |
| $\sum_i w_i \alpha_i$ | 0.036 | 0.034 | 0.033 | 0.033 |
| | [4.55] | [4.58] | [4.23] | [4.00] |
| $\gamma$ | 0.466 | 0.662 | 0.896 | 0.916 |
| | [1.64] | [2.19] | [2.80] | [3.16] |
| Panel adj-$R^2$ (%) | 0.343 | 2.867 | 7.111 | 12.649 |
| $H_0: \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.000 | 0.000 | 0.000 | 0.000 |
| $H_0: \gamma = 0$ | 0.100 | 0.029 | 0.005 | 0.002 |

Table 6: Expected returns in excess of the market of S&P 100 stock portfolios

This Table presents results from regressing portfolio equity returns in excess of the market on the portfolio stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right)$. At the end of every month, we sort S&P 100 firms into decile portfolios based on their CAPM beta, size, book-to-market, or momentum (indicated in the column labels). The return horizon matches the 365 day-maturity of the options used to compute $\mathrm{SVIX}^2_{i,t}$ and $\overline{\mathrm{SVIX}}^2_t$. Panel A reports estimates of the pooled regression specified in equation (15),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (16),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\gamma = 0$. The row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values given in (11). The data is monthly from January 1996 to October 2014.

| Horizon | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.009 | 0.000 | 0.009 | 0.005 |
| | [0.63] | [0.01] | [0.64] | [0.36] |
| $\gamma$ | 0.709 | 1.240 | 0.700 | 0.932 |
| | [1.74] | [7.06] | [1.96] | [2.56] |
| Pooled adj-$R^2$ (%) | 9.278 | 19.399 | 4.896 | 15.588 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.617 | 0.000 | 0.581 | 0.335 |
| $H_0 : \gamma = 0$ | 0.083 | 0.000 | 0.050 | 0.010 |
| Theory adj-$R^2$ (%) | 7.864 | 11.698 | 3.591 | 11.539 |
| *Panel B. Portfolio fixed-effects regressions* | | | | |
| $\sum_i w_i \alpha_i$ | 0.005 | 0.002 | 0.009 | 0.006 |
| | [0.33] | [0.77] | [0.77] | [0.42] |
| $\gamma$ | 0.942 | 1.353 | 0.789 | 1.132 |
| | [1.81] | [5.02] | [1.79] | [2.60] |
| Panel adj-$R^2$ (%) | 11.727 | 21.747 | 8.272 | 18.983 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.649 | 0.000 | 0.465 | 0.227 |
| $H_0 : \gamma = 0$ | 0.071 | 0.000 | 0.074 | 0.009 |

Table 7: Expected returns in excess of the market of S&P 500 stock portfolios

This Table presents results from regressing portfolio equity returns in excess of the market on the portfolio stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right)$. At the end of every month, we sort S&P 500 firms into 25 portfolios based on their CAPM beta, size, book-to-market, or momentum (indicated in the column labels). The return horizon matches the 365 day-maturity of the options used to compute $\mathrm{SVIX}_{i,t}^2$ and $\overline{\mathrm{SVIX}}_t^2$. Panel A reports estimates of the pooled regression specified in equation (15),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (16),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\gamma = 0$. The row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values given in (11). The data is monthly from January 1996 to October 2014.

| Horizon | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.018 | -0.009 | 0.012 | 0.009 |
| | [0.97] | [-0.57] | [0.67] | [0.51] |
| $\gamma$ | 0.396 | 1.543 | 0.656 | 0.757 |
| | [0.97] | [6.29] | [1.13] | [1.69] |
| Pooled adj-$R^2$ (%) | 4.223 | 26.165 | 4.832 | 12.578 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.615 | 0.000 | 0.736 | 0.637 |
| $H_0 : \gamma = 0$ | 0.330 | 0.000 | 0.257 | 0.091 |
| Theory adj-$R^2$ (%) | 3.042 | 13.138 | 3.207 | 10.238 |
| *Panel B. Portfolio fixed-effects regressions* | | | | |
| $\sum_i w_i \alpha_i$ | 0.013 | 0.003 | 0.006 | 0.008 |
| | [0.73] | [1.47] | [0.39] | [0.49] |
| $\gamma$ | 0.676 | 1.700 | 0.683 | 1.022 |
| | [1.05] | [4.64] | [1.10] | [1.68] |
| Panel adj-$R^2$ (%) | 8.533 | 29.979 | 10.357 | 17.267 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.693 | 0.000 | 0.869 | 0.500 |
| $H_0 : \gamma = 0$ | 0.293 | 0.000 | 0.270 | 0.094 |

Table 8: The relationship between realized, expected, and unexpected returns and characteristics

This Table presents results from regressing realized, expected, or unexpected equity returns in excess of the market ($y_{i,t+1}$) on the firm's CAPM beta, log size, book-to-market, past return, and risk-neutral stock variance measured relative to stocks' average risk-neutral variance, $\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t$:

$$y_{i,t+1} = a + b_1 \text{Beta}_{i,t} + b_2 \log(\text{Size}_{i,t}) + b_3 \text{B/M}_{i,t} + b_4 \text{Ret}^{(12,1)}_{i,t} + c\left(\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

The data is monthly and covers S&P 500 firms from January 1996 to October 2014. The first two columns present results for realized returns, the middle two columns for expected returns, and the last two columns for unexpected returns. In columns labelled 'theory,' we set the parameter values of our model forecast to the values implied by equation (11); in columns labelled 'estimated,' we use parameter estimates of a pooled regression (i.e. we use the estimates obtained from the regression specified (15) reported in Table 5). The return horizon is one year. Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. The last three rows report the regression's adjusted-$R^2$ and the $p$-values of Wald tests on joint parameter significance, testing whether all non-constant coefficients are zero or whether all $b_i$-estimates are zero and $c = 0.5$.

| | Realized returns | | Expected returns | | Unexpected returns | |
|---|---|---|---|---|---|---|
| | | | estimated | theory | estimated | theory |
| const | 0.426 | 0.275 | 0.131 | 0.107 | 0.295 | 0.319 |
| | [1.15] | [0.73] | [1.80] | [4.01] | [0.81] | [0.89] |
| $\text{Beta}_{i,t}$ | 0.017 | -0.130 | 0.114 | 0.105 | -0.097 | -0.088 |
| | [0.23] | [-2.08] | [1.74] | [6.37] | [-2.09] | [-1.12] |
| $\log(\text{Size}_{i,t})$ | -0.018 | -0.006 | -0.009 | -0.009 | -0.009 | -0.010 |
| | [-1.31] | [-0.44] | [-1.63] | [-5.42] | [-0.62] | [-0.75] |
| $\text{B/M}_{i,t}$ | 0.032 | 0.031 | 0.001 | 0.001 | 0.031 | 0.031 |
| | [1.28] | [1.16] | [0.10] | [0.11] | [1.19] | [1.21] |
| $\text{Ret}^{(12,1)}_{i,t}$ | -0.051 | -0.029 | -0.017 | -0.015 | -0.034 | -0.035 |
| | [-1.24] | [-0.71] | [-0.95] | [-1.50] | [-0.87] | [-0.89] |
| $\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t$ | | 0.705 | | | | |
| | | [2.30] | | | | |
| adj-$R^2$ (%) | 1.029 | 3.985 | 37.617 | 37.617 | 1.044 | 0.964 |
| $H_0: b_i = 0, c = 0$ | 0.349 | 0.019 | 0.427 | 0.000 | 0.161 | 0.623 |
| $H_0: b_i = 0, c = 0.5$ | | 0.240 | | | | |

Table 9: Out-of-sample forecast accuracy

This Table presents results on the out-of-sample accuracy of our model relative to benchmark predictions. To compare the forecast accuracy of the model to that of the benchmarks, we compute an out-of-sample R-squared, defined as

$$R^2_{OS} = 1 - \frac{\sum_i \sum_t FE^2_M}{\sum_i \sum_t FE^2_B},$$

where $FE_M$ and $FE_B$ denoted the forecast errors from our model and a benchmark prediction, respectively. Panel A evaluates forecasts of expected equity excess returns, as given in equation (12), and Panel B evaluates forecasts of expected equity returns in excess of the market return, as given in equation (11). The data is monthly and covers S&P 500 stocks from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to one year. The return horizons match the maturities of the options used to compute $\text{SVIX}^2_t$, $\text{SVIX}^2_{i,t}$, and $\overline{\text{SVIX}}^2_t$. For Panel A, the benchmark forecasts are the risk-neutral market variance ($\text{SVIX}^2_t$), the time-$t$ historical average excess returns of the S&P 500 ($\overline{\text{S\&P500}}_t$) and the CRSP value-weighted index ($\overline{\text{CRSP}}_t$), a constant prediction of 6% *p.a.*, the stock's risk-neutral variance ($\text{SVIX}^2_{i,t}$), the time-$t$ historical average of the firms' stock excess returns ($\overline{\text{RX}}_{i,t}$), and conditional CAPM implied predictions, where we estimate the CAPM betas from historical return data. For Panel B, we use $\text{SVIX}^2_{i,t}$, a random walk (i.e., zero return forecast), and the conditional CAPM as benchmarks.

*Panel A. Expected excess returns*

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| $\text{SVIX}^2_t$ | 0.00 | 0.49 | 1.72 | 3.10 |
| $\overline{\text{S\&P500}}_t$ | -0.01 | 0.70 | 2.51 | 3.83 |
| $\overline{\text{CRSP}}_t$ | -0.18 | 0.15 | 1.37 | 1.71 |
| 6% *p.a.* | -0.10 | 0.38 | 1.78 | 2.55 |
| $\text{SVIX}^2_{i,t}$ | 1.06 | 1.99 | 1.64 | 2.15 |
| $\overline{\text{RX}}_{i,t}$ | 1.31 | 4.88 | 11.73 | 27.06 |
| $\widehat{\beta}_{i,t} \times \overline{\text{S\&P500}}_t$ | 0.00 | 0.71 | 2.49 | 3.77 |
| $\widehat{\beta}_{i,t} \times \overline{\text{CRSP}}_t$ | -0.16 | 0.20 | 1.41 | 1.69 |
| $\widehat{\beta}_{i,t} \times \text{SVIX}^2_t$ | -0.04 | 0.39 | 1.54 | 2.89 |
| $\widehat{\beta}_{i,t} \times$ 6% *p.a.* | -0.09 | 0.39 | 1.79 | 2.49 |

*Panel B. Expected returns in excess of the market*

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Random walk | 0.08 | 0.67 | 1.85 | 3.10 |
| $(\widehat{\beta}_{i,t} - 1) \times \overline{\text{S\&P500}}_t$ | 0.09 | 0.70 | 1.90 | 3.11 |
| $(\widehat{\beta}_{i,t} - 1) \times \overline{\text{CRSP}}_t$ | 0.13 | 0.79 | 2.07 | 3.36 |
| $(\widehat{\beta}_{i,t} - 1) \times \text{SVIX}^2_t$ | 0.03 | 0.53 | 1.60 | 2.82 |
| $(\widehat{\beta}_{i,t} - 1) \times$ 6% *p.a.* | 0.11 | 0.74 | 1.97 | 3.21 |

Table 10: Model out-of-sample forecasts vs in-sample benchmark predictions

This Table presents results on the out-of-sample accuracy of our model relative to benchmark predictions that also include in-sample information on returns and/or firm characteristics. To compare the forecast accuracy of the model to that of the benchmarks, we compute an out-of-sample R-squared, defined as

$$R^2_{OS} = 1 - \frac{\sum_i \sum_t FE^2_M}{\sum_i \sum_t FE^2_B},$$

where $FE_M$ and $FE_B$ denoted the forecast errors from our model and a benchmark prediction, respectively. Panel A evaluates forecasts of expected equity excess returns, as given in equation (12), and Panel B evaluates forecasts of expected equity returns in excess of the market return, as given in equation (11). The data is monthly and covers S&P 500 stocks from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to one year. The return horizons match the maturities of the options used to compute $\text{SVIX}^2_t$, $\text{SVIX}^2_{i,t}$, and $\overline{\text{SVIX}}^2_t$. For Panel A, the benchmark forecasts are the in-sample average market excess return, a conditional CAPM forecast that uses the in-sample average market excess return as an estimate of the equity premium, the in-sample average return across all stocks; and the fitted values of predictive in-sample regressions of stock returns in excess of the market on CAPM betas, log market capitalization, book-to-market ratios, stock momentum, and all four firm characteristics. For Panel B, we use analogous predictions based on returns in excess of the market.

*Panel A. Expected excess returns*

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| in-sample avg mkt | -0.14 | 0.23 | 1.46 | 1.90 |
| in-sample avg all stocks | -0.19 | 0.09 | 1.20 | 1.42 |
| $\widehat{\beta}_{i,t}\times$ in-sample avg mkt | -0.12 | 0.26 | 1.48 | 1.87 |
| $\text{Beta}_{i,t}$ | -0.19 | 0.08 | 1.17 | 1.30 |
| $\log(\text{Size}_{i,t})$ | -0.28 | -0.25 | 0.57 | 0.21 |
| $\text{B/M}_{i,t}$ | -0.28 | -0.12 | 0.83 | 0.77 |
| $\text{Ret}^{(12,1)}_{i,t}$ | -0.19 | 0.07 | 1.03 | 1.05 |
| All | -0.34 | -0.39 | 0.20 | -0.53 |

*Panel B. Expected returns in excess of the market*

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| in-sample avg all stocks | 0.02 | 0.49 | 1.53 | 2.51 |
| $(\widehat{\beta}_{i,t}-1)\times$ in-sample avg mkt | 0.12 | 0.77 | 2.03 | 3.31 |
| $\text{Beta}_{i,t}$ | 0.02 | 0.49 | 1.53 | 2.48 |
| $\log(\text{Size}_{i,t})$ | -0.03 | 0.30 | 1.20 | 1.93 |
| $\text{B/M}_{i,t}$ | -0.01 | 0.40 | 1.40 | 2.34 |
| $\text{Ret}^{(12,1)}_{i,t}$ | 0.02 | 0.47 | 1.39 | 2.07 |
| All | -0.05 | 0.25 | 1.04 | 1.49 |

Table 11: Portfolio performance with monthly rebalancing

This Table summarizes the performance of monthly rebalanced portfolios invested in S&P 500 stocks from January 1996 to October 2014. We compare the performance of a portfolio based on model-implied expected excess returns to the market benchmark (a value-weighted portfolio) and a naive diversification strategy (an equally-weighted portfolio). For the model portfolio, we apply a rank weighting scheme,

$$w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t \, R_{i,t+1}]^{\theta}}{\sum_i \text{rank}[\mathbb{E}_t \, R_{i,t+1}]^{\theta}},$$

which assigns portfolio weights $w_{i,t}^{XS}$ that increase in expected excess returns, does not allow for short positions, and ensures that the investor is fully invested in the stock market, i.e. $\sum_i w_{i,t}^{XS} = 1$. The parameter $\theta$ controls the aggressiveness of the strategy, and we set $\theta = 2$. For the model forecast-based portfolios, we report annualized average excess returns along with their associated standard deviations and Sharpe ratios as well as their skewness and excess kurtosis. For the benchmark portfolios we additionally report the performance fee (following Fleming et al., 2001) that a risk-averse investor with relative risk aversion $\rho \in \{1, 3, 10\}$ would be willing to pay to switch from the benchmark strategy to the model portfolio. The column labels indicate the variance horizon used in the model forecast.

| | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Portfolio performance based on model forecasts | | | | |
| Mean (%*p.a.*) | 10.85 | 11.67 | 11.86 | 12.53 |
| Sd (%*p.a.*) | 24.50 | 24.57 | 24.63 | 24.60 |
| Sharpe ratio (*p.a.*) | 0.44 | 0.47 | 0.48 | 0.51 |
| Skewness | −0.12 | −0.07 | −0.07 | −0.06 |
| Excess kurtosis | 1.58 | 1.51 | 1.58 | 1.48 |
| Comparison to value-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.45 | 0.44 | 0.43 | 0.41 |
| Skewness | −0.62 | −0.60 | −0.59 | −0.58 |
| Excess kurtosis | 0.83 | 0.77 | 0.74 | 0.68 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 3.07 | 3.93 | 4.28 | 5.20 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 2.61 | 3.47 | 3.82 | 4.75 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 2.32 | 3.17 | 3.52 | 4.46 |
| Comparison to equally-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.54 | 0.56 | 0.55 | 0.55 |
| Skewness | −0.48 | −0.46 | −0.46 | −0.45 |
| Excess kurtosis | 1.64 | 1.59 | 1.63 | 1.55 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 0.66 | 1.20 | 1.47 | 2.13 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 0.29 | 0.83 | 1.10 | 1.76 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 0.05 | 0.59 | 0.86 | 1.52 |

Table 12: Portfolio performance with daily rebalancing

This Table summarizes the performance of daily rebalanced portfolios invested in S&P 500 stocks from January 1996 to October 2014. We compare the performance of a portfolio based on model-implied expected excess returns to the market benchmark (a value-weighted portfolio) and a naive diversification strategy (an equally-weighted portfolio). For the model portfolio, we apply a rank weighting scheme,

$$w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t\, R_{i,t+1}]^{\theta}}{\sum_i \text{rank}[\mathbb{E}_t\, R_{i,t+1}]^{\theta}},$$

which assigns portfolio weights $w_{i,t}^{XS}$ that increase in expected excess returns, does not allow for short positions, and ensures that the investor is fully invested in the stock market, i.e. $\sum_i w_{i,t}^{XS} = 1$. The parameter $\theta$ controls the aggressiveness of the strategy, and we set $\theta = 2$. For the model forecast-based portfolios, we report annualized average excess returns along with their associated standard deviations and Sharpe ratios as well as their skewness and excess kurtosis. For the benchmark portfolios we additionally report the performance fee (following Fleming et al., 2001) that a risk-averse investor with relative risk aversion $\rho \in \{1, 3, 10\}$ would be willing to pay to switch from the benchmark strategy to the model portfolio. The column labels indicate the variance horizon used in the model forecast.

| | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Portfolio performance based on model forecasts | | | | |
| Mean (%*p.a.*) | 13.86 | 13.49 | 13.38 | 13.49 |
| Sd (%*p.a.*) | 27.70 | 27.78 | 27.76 | 27.59 |
| Sharpe ratio (*p.a.*) | 0.50 | 0.49 | 0.48 | 0.49 |
| Skewness | −0.03 | −0.04 | −0.04 | −0.04 |
| Excess kurtosis | 7.66 | 7.47 | 7.39 | 7.27 |
| Comparison to value-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.38 | 0.38 | 0.38 | 0.38 |
| Skewness | −0.18 | −0.18 | −0.18 | −0.18 |
| Excess kurtosis | 6.22 | 6.21 | 6.22 | 6.22 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 5.41 | 5.04 | 4.93 | 5.07 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 4.94 | 4.56 | 4.45 | 4.61 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 4.63 | 4.24 | 4.14 | 4.30 |
| Comparison to equally-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.51 | 0.51 | 0.51 | 0.52 |
| Skewness | −0.21 | −0.21 | −0.21 | −0.21 |
| Excess kurtosis | 7.29 | 7.29 | 7.29 | 7.30 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 2.42 | 2.04 | 1.94 | 2.03 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 1.98 | 1.60 | 1.49 | 1.60 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 1.71 | 1.33 | 1.23 | 1.34 |

Table 13: Performance of size/value-portfolios with daily rebalancing

This Table summarizes the performance of daily rebalanced portfolios invested in a subset of S&P 500 stocks, conditional on firm size and book-to-market. At every portfolio rebalancing date $t$, we classify firms as small (big) when their market capitalization is in the bottom (top) tertile of the time-$t$ distribution across all firms in our sample. Similarly, we classify firms as value (growth) stocks when their book-to-market ratio is within the top (bottom) tertile of the book-to-market distribution at time $t$. We report results for firms that are classified as small value, small growth, big value, and big growth. The sample period is from January 1996 to October 2014. We compare the performance of a portfolio based on model-implied expected excess returns (based on a variance horizon of one year) to a value-weighted portfolio and a naive diversification strategy (an equally-weighted portfolio). For the model portfolio, we apply a rank weighting scheme,

$$w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t\, R_{i,t+1}]^{\theta}}{\sum_i \text{rank}[\mathbb{E}_t\, R_{i,t+1}]^{\theta}},$$

which assigns portfolio weights $w_{i,t}^{XS}$ that increase in expected excess returns, does not allow for short positions, and ensures that the investor is fully invested in the stock market, i.e. $\sum_i w_{i,t}^{XS} = 1$. The parameter $\theta$ controls the aggressiveness of the strategy, and we set $\theta = 2$. For the model forecast-based portfolios, we report annualized average excess returns along with their associated standard deviations and Sharpe ratios as well as their skewness and excess kurtosis. For the benchmark portfolios we additionally report the performance fee (following Fleming et al., 2001) that a risk-averse investor with relative risk aversion $\rho \in \{1, 3, 10\}$ would be willing to pay to switch from the benchmark strategy to the model portfolio.

| | Small Value | Small Growth | Big Value | Big Growth |
|---|---|---|---|---|
| Portfolio performance based on model forecasts | | | | |
| Mean (%$p.a.$) | 25.52 | 21.24 | 11.71 | 10.06 |
| Sd (%$p.a.$) | 32.52 | 35.49 | 30.41 | 26.81 |
| Sharpe ratio ($p.a.$) | 0.78 | 0.60 | 0.39 | 0.38 |
| Skewness | 0.11 | 0.52 | 0.12 | 0.21 |
| Excess kurtosis | 7.21 | 6.94 | 12.31 | 5.62 |
| Comparison to value-weighted portfolio | | | | |
| Sharpe ratio ($p.a.$) | 0.55 | 0.42 | 0.29 | 0.32 |
| Skewness | −0.22 | 0.25 | −0.11 | −0.02 |
| Excess kurtosis | 7.37 | 6.38 | 7.95 | 4.97 |
| Performance fee with $\rho = 1$ (%$p.a.$) | 11.12 | 9.05 | 3.82 | 2.95 |
| Performance fee with $\rho = 3$ (%$p.a.$) | 11.12 | 9.05 | 3.82 | 2.95 |
| Performance fee with $\rho = 10$ (%$p.a.$) | 10.12 | 7.76 | 3.17 | 2.26 |
| Comparison to equally-weighted portfolio | | | | |
| Sharpe ratio ($p.a.$) | 0.67 | 0.52 | 0.36 | 0.37 |
| Skewness | −0.11 | 0.27 | −0.23 | −0.05 |
| Excess kurtosis | 6.82 | 6.89 | 8.08 | 5.41 |
| Performance fee with $\rho = 1$ (%$p.a.$) | 8.00 | 6.50 | 2.54 | 2.08 |
| Performance fee with $\rho = 3$ (%$p.a.$) | 7.45 | 5.76 | 2.02 | 1.64 |
| Performance fee with $\rho = 10$ (%$p.a.$) | 7.09 | 5.27 | 1.70 | 1.38 |

Figure 3: Option-implied equity variance of S&P 100 firms

This Figure plots the time-series of the risk-neutral variance of the market ($\text{SVIX}_t^2$) and of stocks' average risk-neutral variance ($\overline{\text{SVIX}}_t^2$). We compute $\text{SVIX}_t^2$ from equity index options on the S&P 100. $\overline{\text{SVIX}}_t^2$ is the value-weighted sum of S&P 100 stocks' risk-neutral variance computed from individual firm equity options. Panels A through D present the variance series implied by equity options with maturities of one, three, six, and twelve months, respectively. The data is daily from January 1996 to October 2014.

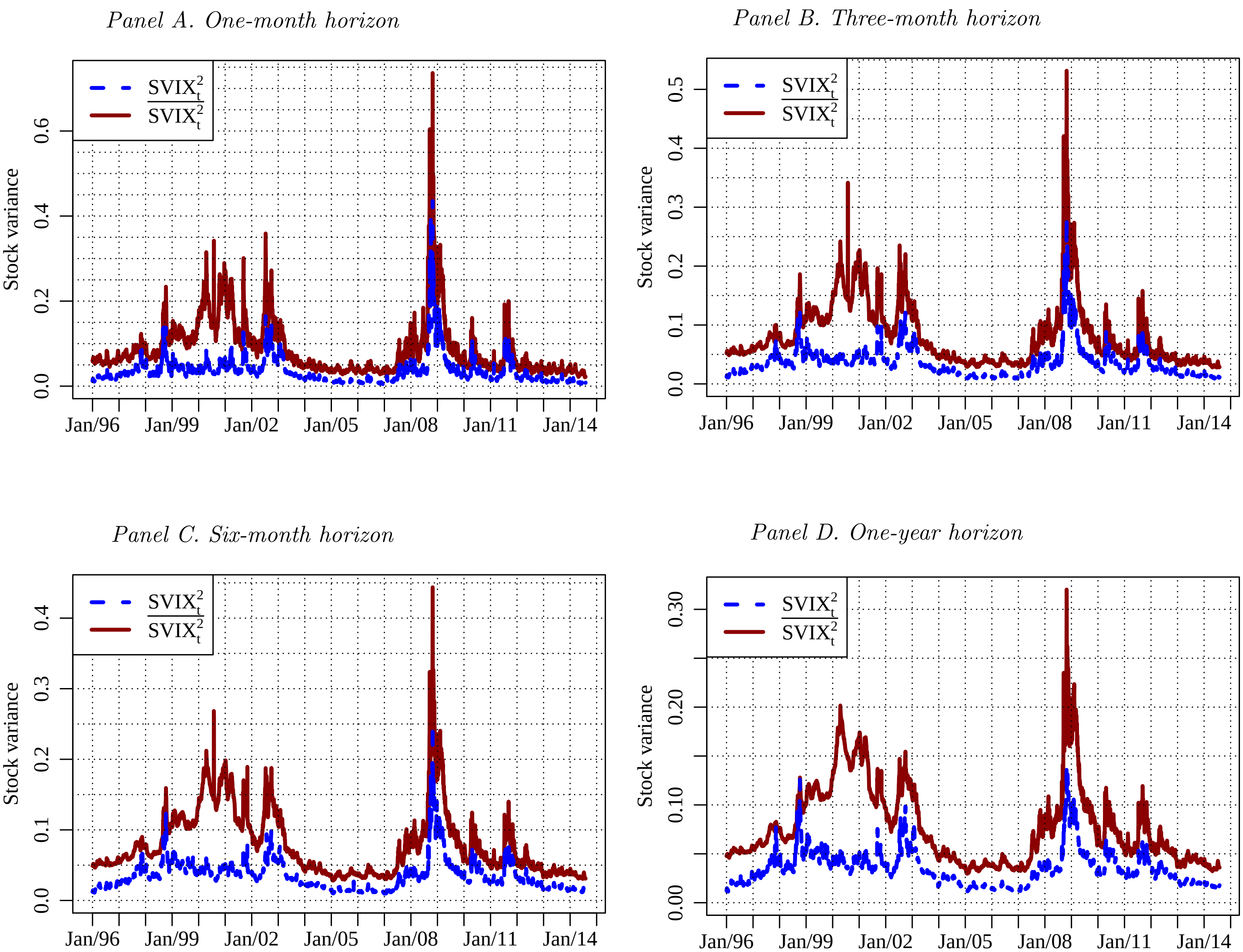
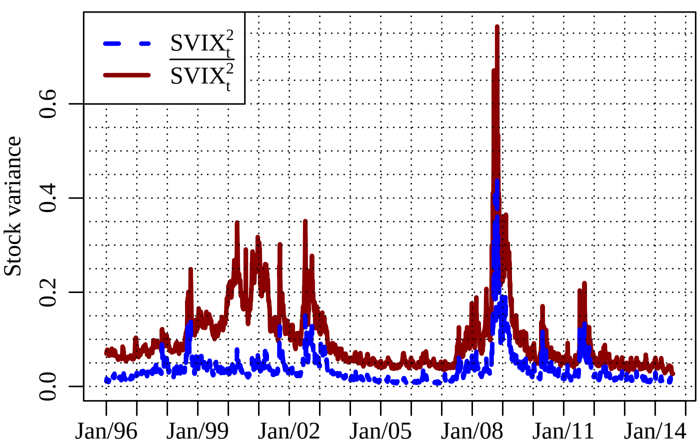


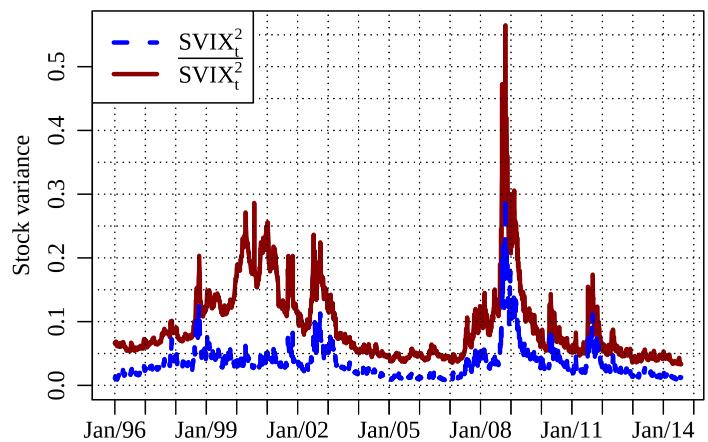
Figure 4: Option-implied equity variance of S&P 500 firms

This Figure plots the time-series of the risk-neutral variance of the market ($\text{SVIX}_t^2$) and of stocks' average risk-neutral variance ($\overline{\text{SVIX}}_t^2$). We compute $\text{SVIX}_t^2$ from equity index options on the S&P 500. $\overline{\text{SVIX}}_t^2$ is the value-weighted sum of S&P 500 stocks' risk-neutral variance computed from individual firm equity options. Panels A through D present the variance series implied by equity options with maturities of one, three, six, and twelve months, respectively. The data is daily from January 1996 to October 2014.

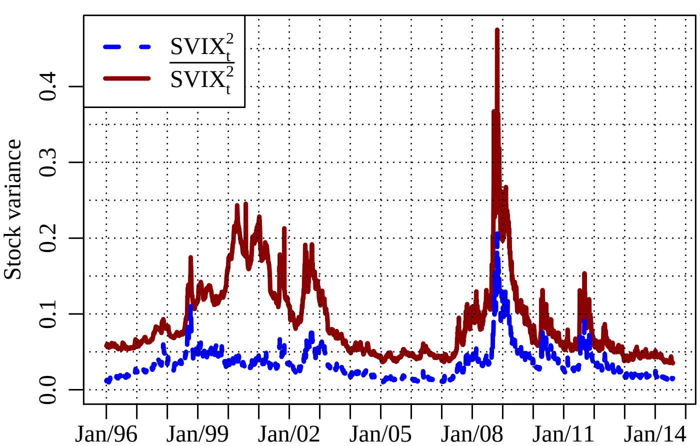
*Panel A. One-month horizon*




*Panel B. Three-month horizon*




*Panel C. Six-month horizon*




*Panel D. One-year horizon*

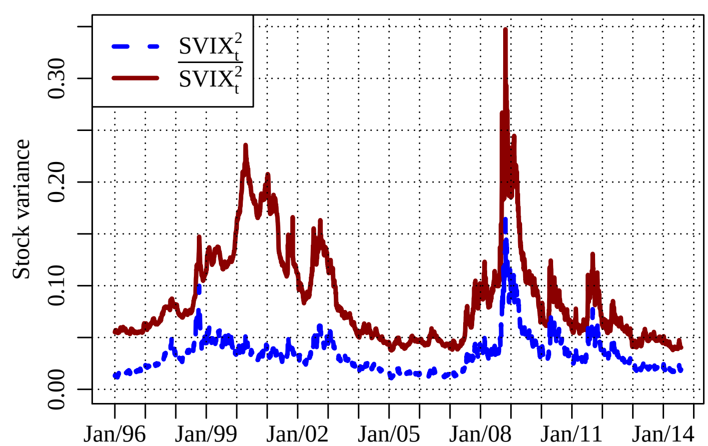


Figure 5: A measure of average risk-neutral correlation between stocks

This Figure plots the time-series of the ratio of the risk-neutral variance of the market to stocks' average risk-neutral variance ($\text{SVIX}_t^2 / \overline{\text{SVIX}}_t^2$); in the appendix, we show that this quantity is an approximate measure of average risk-neutral correlation. We compute $\text{SVIX}_t^2$ from equity index options on the S&P 100 (Panels A and B) and S&P 500 (Panels C and D). $\overline{\text{SVIX}}_t^2$ is the corresponding value-weighted sum of S&P 100 or S&P 500 stocks' risk-neutral variance computed from individual firm equity options. The data is daily from January 1996 to October 2014.

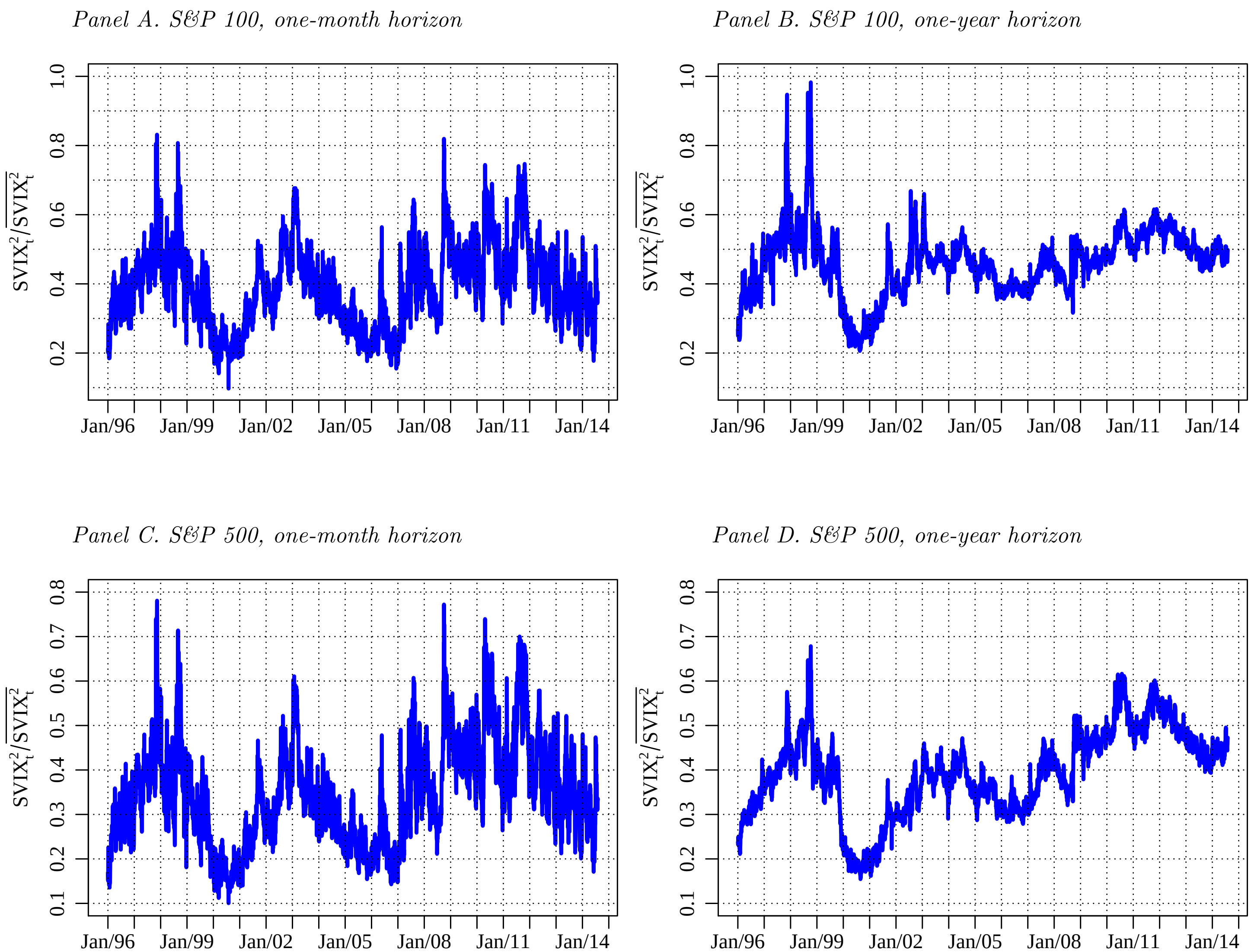


Figure 6: Beta, size, value, momentum, and option-implied equity variance

This Figure reports (equally-weighted) averages of risk-neutral stock variance ($\text{SVIX}^2_{i,t}$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. At every date $t$, we assign stocks to decile portfolios based on on their characteristics and report the time-series averages of $\text{SVIX}^2_{i,t}$ across deciles using $\text{SVIX}^2_{i,t}$-horizons of one year (Panels A to D). The sample period is January 1996 to October 2014.

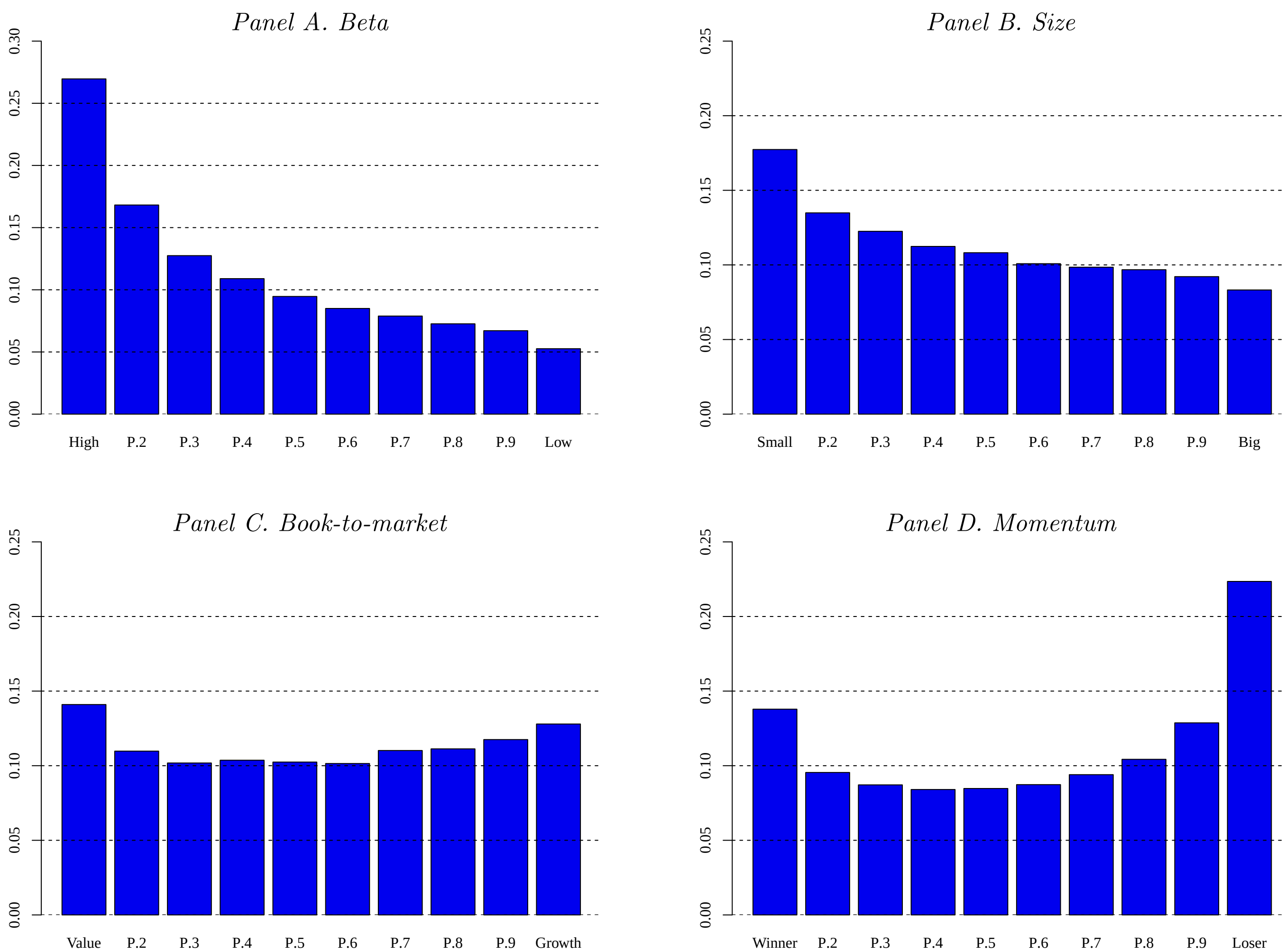


Figure 7: Beta, size, value, momentum, and option-implied equity variance

This Figure plots the time-series of risk-neutral stock variance ($\text{SVIX}^2_{i,t}$) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. The horizon is annual. At every date $t$, we classify firms as small, medium, or big when their market capitalization is in the bottom, middle, or top tertile of the time-$t$ distribution across all firms in our sample, and compute the (equally-weighted) average $\text{SVIX}^2_{i,t}$. Similarly, we classify firms by their other characteristics at time $t$. The sample period is from January 1996 to October 2014.

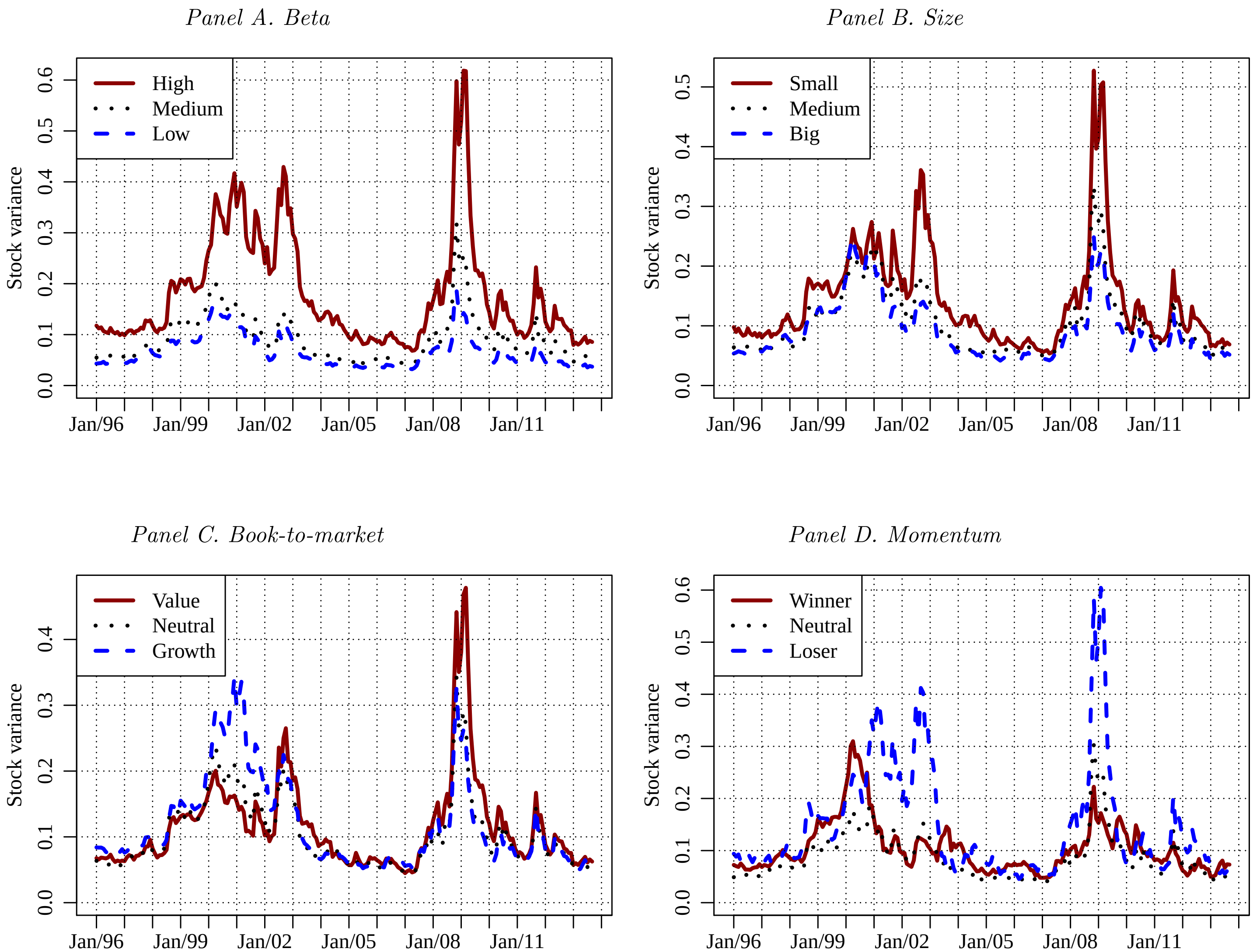


Figure 8: Average equity excess returns

This Figure presents results on the relation between a firm's equity excess returns and its risk-neutral variance measured relative to average risk-neutral stock variance. For firms that were constituents of the S&P 500 index throughout our sample period, we compute time-series averages of their excess returns and their stock volatility relative to stocks' average volatility ($\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$). We multiply the stock variance estimate by 0.5 and plot the pairwise combinations (blue crosses) for horizons of one, three, six, and, twelve months (Panels A to D). The black line represents the regression fit to the individual firm observations with slope coefficient and R-squared reported in the plot legend. Our theory implies that the slope coefficient of this regression should be one and that the intercept corresponds to the average excess return of the market. The red diamonds represent decile portfolios of firms generated from ranking firms by their average time-$t$ decile classification based on $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$ among all S&P 500 firms over our sample period. Similarly, the triangles in orange represent portfolios of stocks formed according to their firms' average size and book-to-market classification.

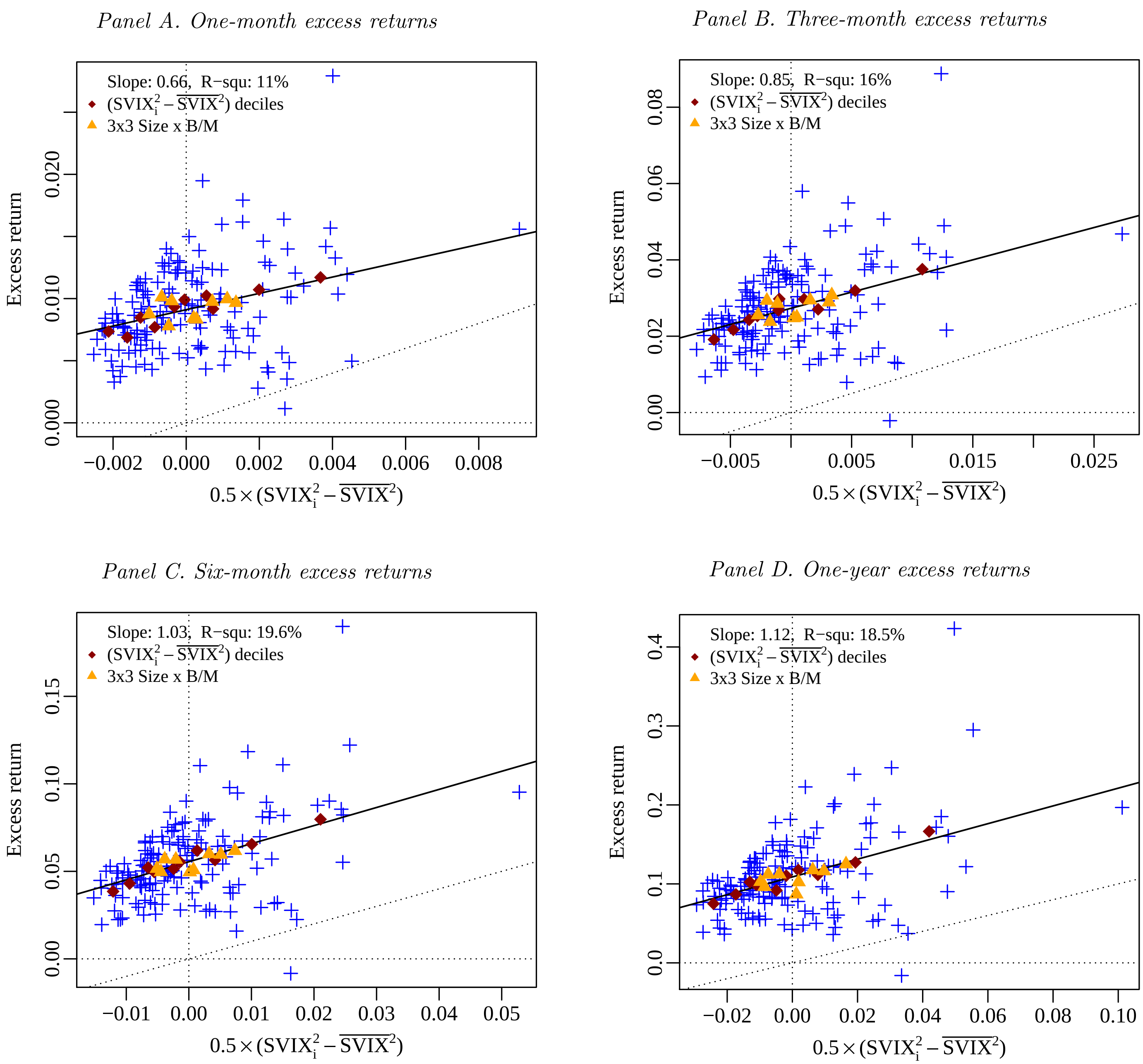
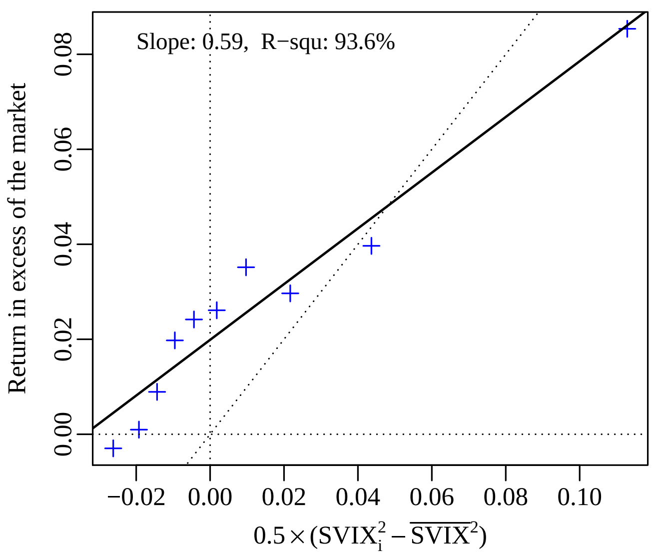


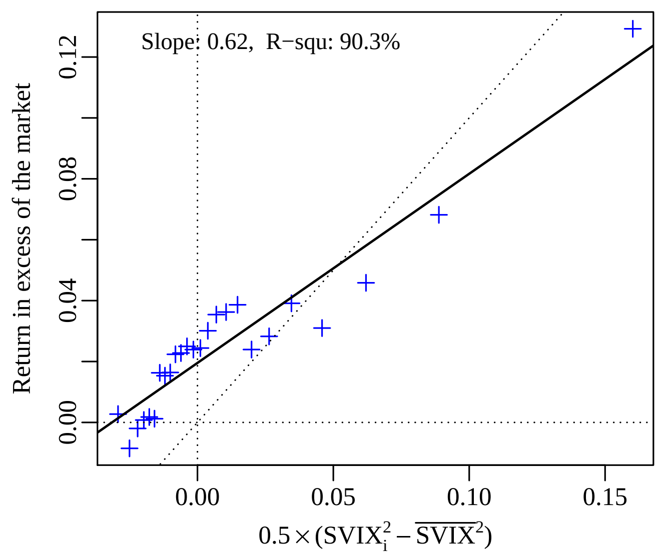
Figure 9: Portfolios sorted by excess stock volatility

This Figure reports results on the relationship between equity portfolio returns in excess of the market and risk-neutral stock variance measured relative to stocks' average risk-neutral variance. At the end of each month, we group all available firms into 10, 25, 50, or 100 portfolios (Panels A to D) based on their stock volatility relative to aggregate stock volatility ($\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$); the horizon is annual. For each portfolio, we compute the time-series average return in excess of the market and plot the pairwise combinations with the corresponding stock variance estimate multiplied by 0.5. Our theory implies that the slope coefficient of this regression should be one. The black line represents the regression fit to the portfolio observations with slope coefficient and R-squared reported in the plot legend. The sample period is January 1996 to October 2014.

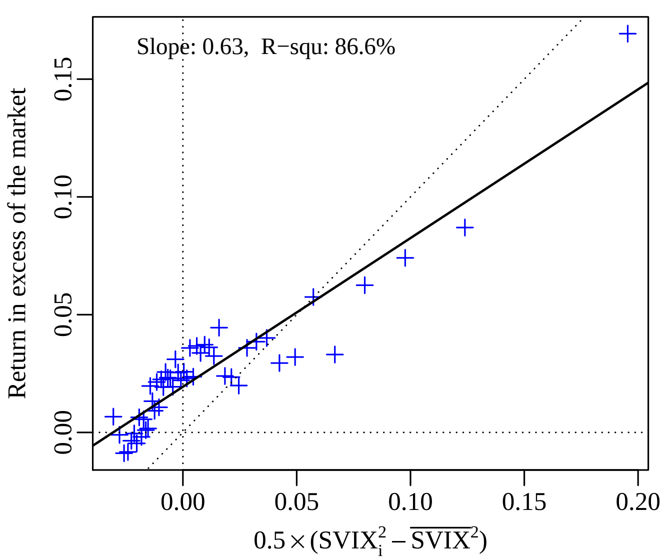
*Panel A. 10 portfolios*




*Panel B. 25 portfolios*




*Panel C. 50 portfolios*




*Panel D. 100 portfolios*

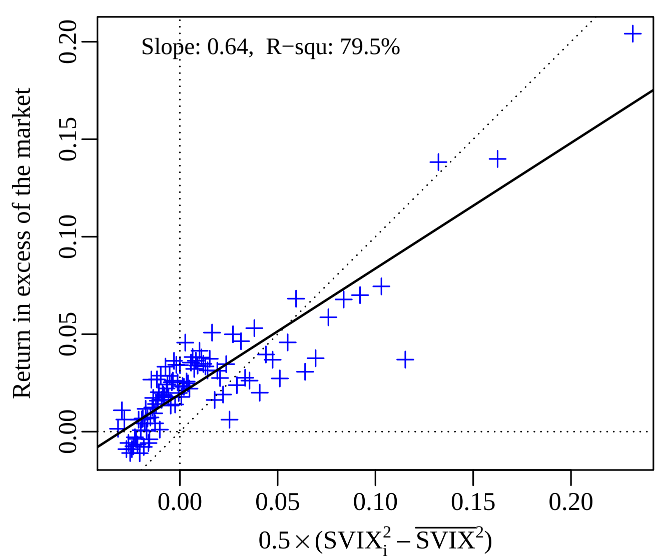


Figure 10: Portfolios sorted by beta, size, book-to-market, and momentum

This Figure reports results on the relationship between equity portfolio returns in excess of the market and risk-neutral stock variance measured relative to average firm-level risk-neutral variance. At the end of each month, we form 25 portfolios based on firms' beta, size, book-to-market, or momentum (Panels A to D). For each portfolio, we compute the time-series average return in excess of the market and plot the pairwise combinations with the corresponding stock variance estimate multiplied by 0.5. The black line represents the regression fit to the portfolio observations with slope coefficient and R-squared reported in the plot legend. Our theory implies that the slope coefficient of this regression should be one. The sample period is January 1996 to October 2014.

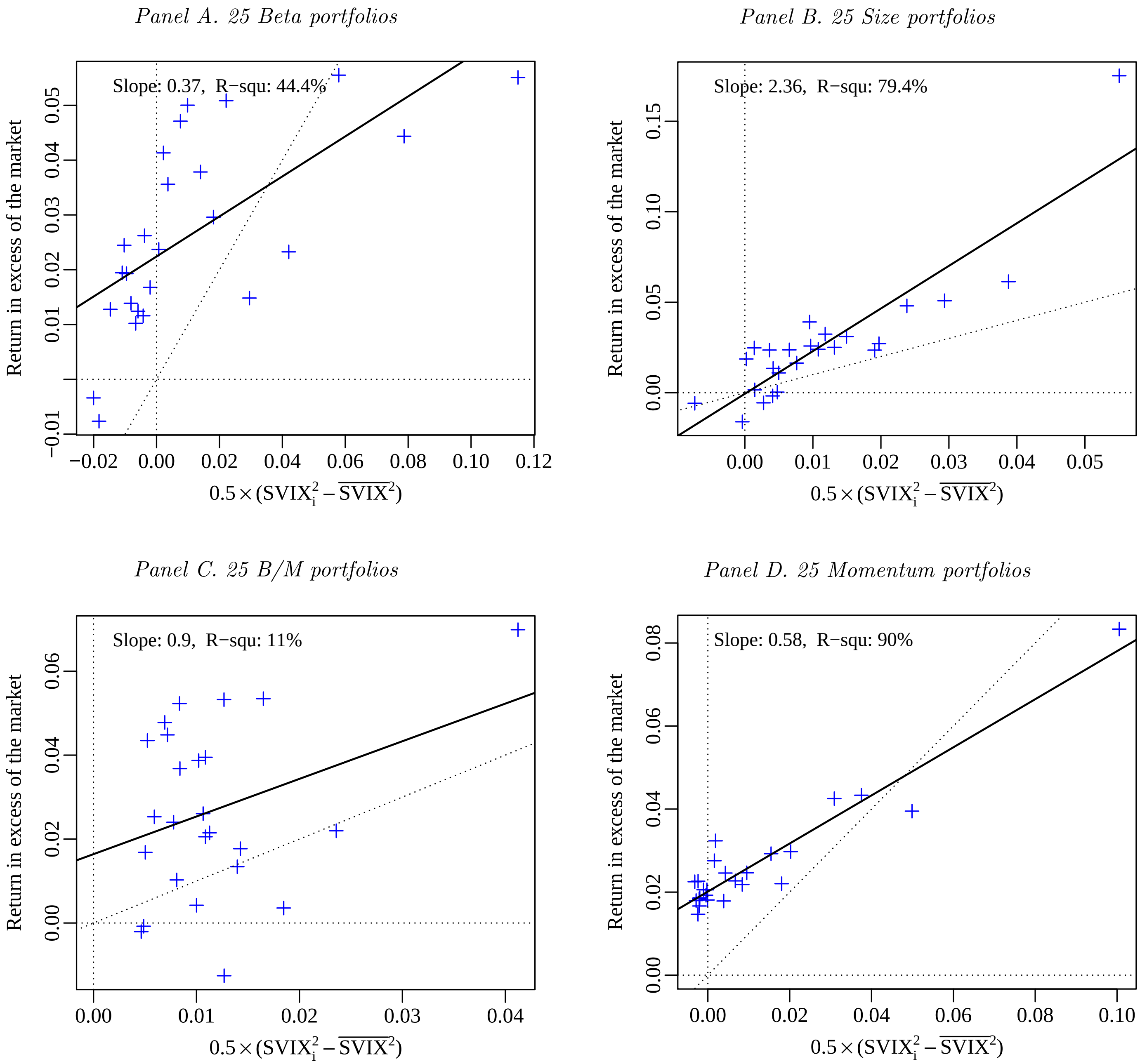
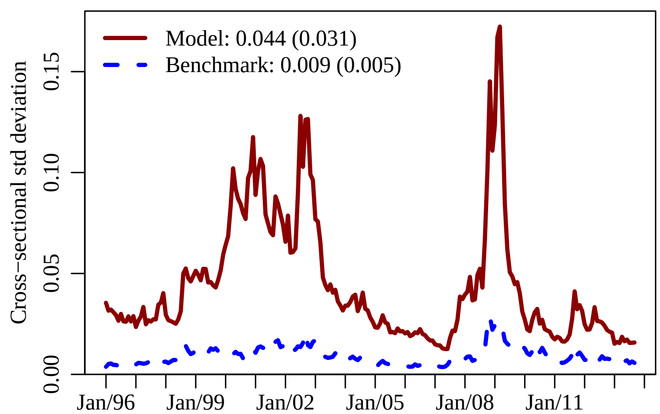


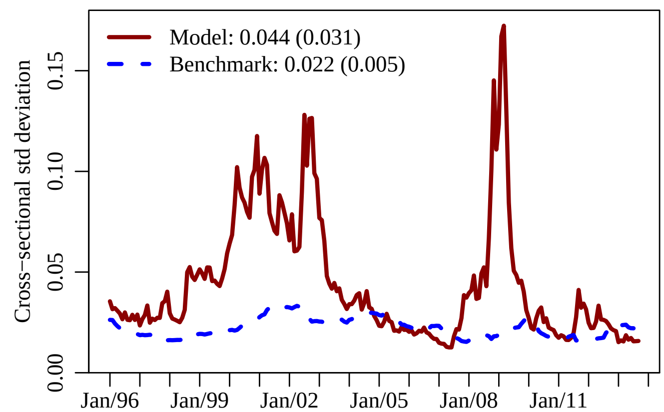
Figure 11: Cross-sectional variation in expected returns

This Figure plots the time-series of the cross-sectional standard deviation of one-year expected excess returns generated by our model and by benchmark predictions. The benchmark predictions are CAPM forecasts using conditional betas (estimated from historical returns) and an estimate of the equity premium. We consider four proxies for the latter: the risk-neutral variance of the market ($\mathrm{SVIX}_t^2$), the historical average excess returns of the S&P 500 ($\overline{\mathrm{S\&P500}}_t$) and of the CRSP value-weighted index ($\overline{\mathrm{CRSP}}_t$), and a constant prediction of 6% *p.a.* The time-series averages of the cross-sectional standard deviations of expected returns are reported in the plot legends, together with time-series standard deviations in parentheses. The data is monthly and covers S&P 500 stocks from January 1996 to October 2014.

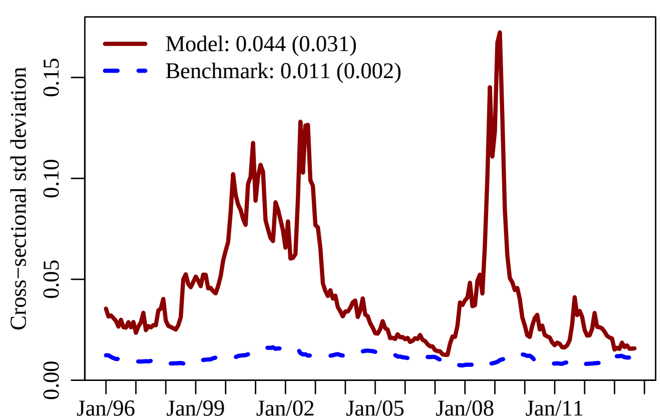
*Panel A.* $\mathrm{SVIX}_t^2$*-CAPM forecasts*




*Panel B.* $\overline{CRSP}_t$*-CAPM forecasts*




*Panel C.* $\overline{S\&P500}_t$ *-CAPM forecasts*




*Panel D. 6%-CAPM forecast*

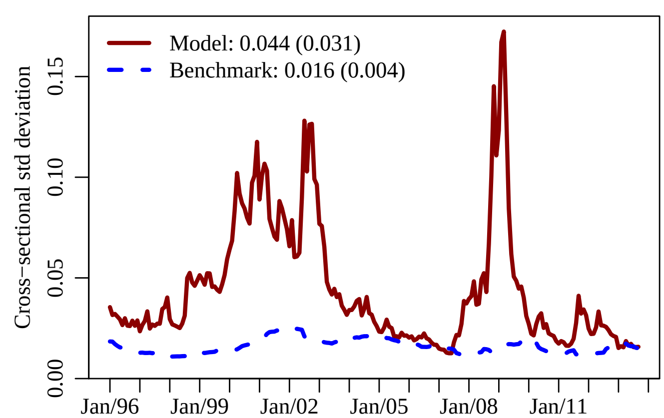
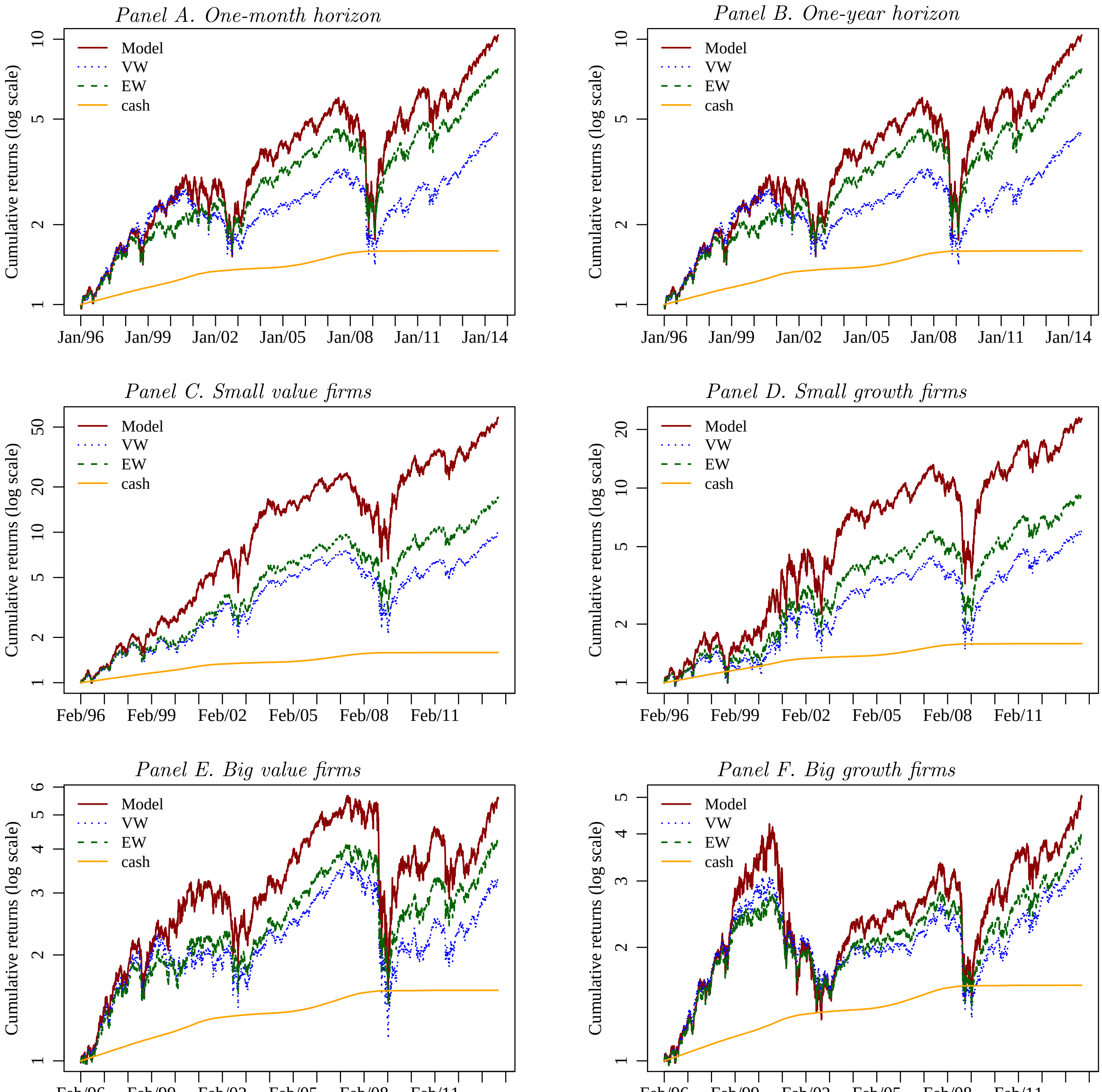


Figure 12: Cumulative portfolio returns

This Figure plots the cumulative returns of daily rebalanced portfolios invested in S&P 500 stocks from January 1996 to October 2014. We compare the performance of a portfolio based on model-implied expected returns to the market benchmark (a value-weighted portfolio) and a naive diversification strategy (an equally-weighted portfolio). For the model portfolio, we apply a rank weighting scheme, $w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t\, R_{i,t+1}]^\theta}{\sum_i \text{rank}[\mathbb{E}_t\, R_{i,t+1}]^\theta}$, which assigns portfolio weights $w_{i,t}^{XS}$ that increase in expected excess returns, does not allow for short positions, and ensures that the investor is fully invested in the stock market, i.e. $\sum_i w_{i,t}^{XS} = 1$. The parameter $\theta$ controls the aggressiveness of the strategy; we set $\theta = 2$. Panels A and B plot the cumulative portfolio returns when using forecasts based on one-month and one-year variance horizons, respectively. Panels C to F plot the cumulative portfolio returns for subsets of firms based on their size and book-to-market ratios with forecasts based on a one-year variance horizon.



Internet Appendix for

# What is the Expected Return on a Stock?

Table IA.1: Expected excess returns of S&P 100 stock portfolios

This Table presents results from regressing equity portfolio excess returns on the risk-neutral variance of the market variance ($\mathrm{SVIX}_t^2$) and the stock portfolios' firm-level risk-neutral variance measured relative to stocks' average risk-neutral variance ($\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2$). At the end of each month, we sort S&P 100 firms into decile portfolios based on their CAPM beta, size, book-to-market, or momentum. The horizon is annual. Panel A reports estimates of the pooled regression specified in equation (13),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\,\mathrm{SVIX}_t^2 + \gamma\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with portfolio fixed effects specified in equation (14),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\,\mathrm{SVIX}_t^2 + \gamma\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_{i,t}\alpha_i$ is the time-series average of the value-weighted sum of portfolio fixed effects. Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept, $\beta = 1$ and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\beta = \gamma = 0$. The 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when fixing the parameters at their theoretical values. The data is monthly from January 1996 to October 2014.

| Horizon | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | -0.003 | 0.000 | -0.002 | -0.001 |
| | [-0.04] | [0.00] | [-0.03] | [-0.02] |
| $\beta$ | 2.004 | 1.659 | 1.974 | 1.854 |
| | [1.41] | [1.26] | [1.34] | [1.38] |
| $\gamma$ | 0.742 | 1.326 | 0.787 | 0.979 |
| | [2.18] | [5.28] | [2.50] | [2.94] |
| Pooled adj-$R^2$ (%) | 8.252 | 12.615 | 6.393 | 11.095 |
| $H_0 : \alpha = 0, \beta = 1, \gamma = 0.5$ | 0.648 | 0.009 | 0.627 | 0.539 |
| $H_0 : \beta = \gamma = 0$ | 0.078 | 0.000 | 0.030 | 0.013 |
| Theory adj-$R^2$ (%) | 4.333 | 5.439 | 2.050 | 5.723 |
| *Panel B. Portfolio fixed-effects regressions* | | | | |
| $\sum_i w_i\alpha_i$ | -0.002 | 0.007 | -0.000 | 0.004 |
| | [-0.03] | [0.10] | [-0.00] | [0.06] |
| $\beta$ | 1.844 | 1.563 | 1.910 | 1.720 |
| | [1.35] | [1.22] | [1.31] | [1.32] |
| $\gamma$ | 1.018 | 1.489 | 0.897 | 1.206 |
| | [2.72] | [4.05] | [2.40] | [3.22] |
| Panel adj-$R^2$ (%) | 18.765 | 23.485 | 18.615 | 22.128 |
| $H_0 : \sum_i w_i\alpha_i = 0, \beta = 1, \gamma = 0.5$ | 0.538 | 0.052 | 0.603 | 0.267 |
| $H_0 : \beta = \gamma = 0$ | 0.025 | 0.000 | 0.043 | 0.005 |

## Table IA.2: Expected excess returns of S&P 500 stock portfolios

This Table presents results from regressing equity portfolio excess returns on the risk-neutral variance of the market variance ($\text{SVIX}_t^2$) and the stock portfolios' firm-level risk-neutral variance measured relative to stocks' average risk-neutral variance ($\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$). At the end of each month, we sort S&P 500 firms into decile portfolios based on their CAPM beta, size, book-to-market, or momentum. The horizon is annual. Panel A reports estimates of the pooled regression specified in equation (13),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\, \text{SVIX}_t^2 + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with portfolio fixed effects specified in equation (14),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\, \text{SVIX}_t^2 + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_{i,t}\alpha_i$ is the time-series average of the value-weighted sum of portfolio fixed effects. Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept, $\beta = 1$ and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\beta = \gamma = 0$. The 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when fixing the parameters at their theoretical values. The data is monthly from January 1996 to October 2014.

| Horizon | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | -0.020 | -0.018 | -0.021 | -0.020 |
| | [-0.29] | [-0.26] | [-0.30] | [-0.29] |
| $\beta$ | 3.078 | 2.271 | 3.064 | 2.840 |
| | [1.86] | [1.57] | [1.81] | [1.80] |
| $\gamma$ | 0.326 | 1.414 | 0.399 | 0.679 |
| | [0.90] | [3.52] | [0.95] | [1.57] |
| Pooled adj-$R^2$ (%) | 7.499 | 15.306 | 8.120 | 10.847 |
| $H_0: \alpha = 0, \beta = 1, \gamma = 0.5$ | 0.065 | 0.143 | 0.081 | 0.277 |
| $H_0: \beta = \gamma = 0$ | 0.166 | 0.002 | 0.183 | 0.130 |
| Theory adj-$R^2$ (%) | 1.108 | 5.193 | 0.857 | 4.468 |
| *Panel B. Portfolio fixed-effects regressions* | | | | |
| $\sum_i w_i\alpha_i$ | -0.018 | -0.006 | -0.027 | -0.015 |
| | [-0.27] | [-0.08] | [-0.38] | [-0.23] |
| $\beta$ | 2.924 | 2.181 | 3.070 | 2.658 |
| | [1.83] | [1.52] | [1.82] | [1.74] |
| $\gamma$ | 0.527 | 1.530 | 0.392 | 0.912 |
| | [1.03] | [2.48] | [0.91] | [1.59] |
| Panel adj-$R^2$ (%) | 19.920 | 27.481 | 22.470 | 23.049 |
| $H_0: \sum_i w_i\alpha_i = 0, \beta = 1, \gamma = 0.5$ | 0.110 | 0.359 | 0.105 | 0.384 |
| $H_0: \beta = \gamma = 0$ | 0.186 | 0.030 | 0.182 | 0.138 |

Table IA.3: Excess market returns of characteristics/$\text{SVIX}_{i,t}$ double-sorted portfolios

This Table presents results from regressing portfolio equity returns in excess of the market on the portfolio stock's risk-neutral variance measured relative to stocks' average risk-neutral variance, $\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t$. At the end of each month, we sort S&P 500 firms into 5x5-double sorted portfolios based on firm characteristics and $\text{SVIX}_{i,t}$. We first assign firms to quintile portfolios based on their CAPM beta, size, book-to-market, or momentum. In the second step, we sort stocks within each of the characteristics portfolios into $\text{SVIX}_{i,t}$-quintiles, providing us with a total of 25 conditionally double-sorted portfolios. The horizon of the portfolio returns matches the 365 day-maturity of the options used to compute $\text{SVIX}^2_{i,t}$ and $\overline{\text{SVIX}}^2_t$. Panel A reports estimates of the pooled regression specified in equation (15),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left( \text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t \right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (16),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left( \text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t \right) + \varepsilon_{i,t+1}.$$

Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\gamma = 0$. The 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when fixing the parameters at their theoretical values. The data is monthly from January 1996 to October 2014.

| Horizon | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.015 | 0.013 | 0.015 | 0.014 |
| | [0.78] | [0.68] | [0.78] | [0.71] |
| $\gamma$ | 0.497 | 0.575 | 0.503 | 0.564 |
| | [1.59] | [1.78] | [1.54] | [1.77] |
| Pooled adj-$R^2$ (%) | 8.473 | 10.032 | 8.175 | 10.426 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.634 | 0.590 | 0.637 | 0.601 |
| $H_0 : \gamma = 0$ | 0.111 | 0.074 | 0.124 | 0.076 |
| Theory adj-$R^2$ (%) | 7.682 | 9.107 | 7.316 | 9.514 |
| *Panel B. Portfolio fixed-effects regressions* | | | | |
| $\sum_i w_i \alpha_i$ | 0.015 | 0.008 | 0.014 | 0.019 |
| | [0.87] | [1.58] | [0.89] | [1.10] |
| $\gamma$ | 0.793 | 0.949 | 0.714 | 0.871 |
| | [1.61] | [1.80] | [1.41] | [1.78] |
| Panel adj-$R^2$ (%) | 13.073 | 16.579 | 12.728 | 15.240 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.445 | 0.069 | 0.481 | 0.208 |
| $H_0 : \gamma = 0$ | 0.107 | 0.073 | 0.160 | 0.075 |

Table IA.4: Excess market returns of $\mathrm{SVIX}_{i,t}$/characteristics double-sorted portfolios

This Table presents results from regressing portfolio equity returns in excess of the market on the portfolio stock's risk-neutral variance measured relative to stocks' average risk-neutral variance, $\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t$. At the end of each month, we sort S&P 500 firms into 5x5-double sorted portfolios based on $\mathrm{SVIX}_{i,t}$ and firm characteristics. We first assign firms to quintile portfolios based on $\mathrm{SVIX}_{i,t}$. In the second step, we sort stocks within each $\mathrm{SVIX}_{i,t}$-portfolio in quintiles based on their CAPM beta, size, book-to-market, or momentum, providing us with a total of 25 conditionally double-sorted portfolios. The horizon is annual. Panel A reports estimates of the pooled regression specified in equation (15),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (16),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. In each panel, we report $p$-values of Wald tests of two hypotheses: first, that the parameters take the values predicted by our theory (zero intercept and $\gamma = 0.5$); and, second, the uninteresting null hypothesis that $\gamma = 0$. The 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when fixing the parameters at their theoretical values. The data is monthly from January 1996 to October 2014.

| Horizon | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled regressions* | | | | |
| $\alpha$ | 0.016 | 0.014 | 0.015 | 0.014 |
| | [0.82] | [0.69] | [0.77] | [0.71] |
| $\gamma$ | 0.456 | 0.568 | 0.509 | 0.555 |
| | [1.37] | [1.70] | [1.55] | [1.63] |
| Pooled adj-$R^2$ (%) | 6.371 | 9.072 | 8.049 | 9.294 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.648 | 0.591 | 0.633 | 0.614 |
| $H_0 : \gamma = 0$ | 0.170 | 0.088 | 0.120 | 0.103 |
| Theory adj-$R^2$ (%) | 5.516 | 8.166 | 7.178 | 8.416 |
| *Panel B. Portfolio fixed-effects regressions* | | | | |
| $\sum_i w_i \alpha_i$ | 0.016 | 0.007 | 0.015 | 0.019 |
| | [0.96] | [0.95] | [0.90] | [1.07] |
| $\gamma$ | 0.802 | 0.971 | 0.850 | 0.963 |
| | [1.49] | [1.76] | [1.61] | [1.74] |
| Panel adj-$R^2$ (%) | 11.225 | 15.440 | 13.465 | 14.605 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.381 | 0.254 | 0.377 | 0.216 |
| $H_0 : \gamma = 0$ | 0.137 | 0.078 | 0.108 | 0.083 |

Table IA.5: The relationship between realized, expected, and unexpected returns and characteristics

This Table presents results from regressing realized, expected, and unexpected equity excess returns on the firm's CAPM beta, log size, book-to-market, past return, risk-neutral market variance ($\mathrm{SVIX}_t$), and risk-neutral stock variance measured relative to stocks' average risk-neutral variance ($\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t$):

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = a + b_1 \mathrm{Beta}_{i,t} + b_2 \log(\mathrm{Size}_{i,t}) + b_3 \mathrm{B/M}_{i,t} + b_4 \mathrm{Ret}^{(12,1)}_{i,t} + c_0\, \mathrm{SVIX}^2_t + c_1 \left( \mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t \right) + \varepsilon_{i,t+1}.$$

The data is monthly and covers S&P 500 firms from January 1996 to October 2014. The first two columns present results for realized returns, the middle two columns for expected returns, and the last two columns for unexpected returns. In columns labelled 'theory', we set the parameter values of our model forecast to the values implied by theory (i.e. we use equation (12)); while in columns labelled 'estimated', we use parameter estimates of a pooled regression (i.e. we use the results reported in Table 3). The horizon is annual. Values in square brackets are $t$-statistics based on standard errors from a block bootstrap procedure. The last three rows report adjusted-$R^2$ and the $p$-values of Wald tests of joint parameter significance, testing (i) whether all non-constant coefficients are jointly zero and (ii) whether all $b_i$-estimates are zero, $c_0 = 1$, and $c = 0.5$.

| | Realized returns | | Expected returns | | Unexpected returns | |
|---|---|---|---|---|---|---|
| | | | estimated | theory | estimated | theory |
| const | 0.721 | 0.453 | 0.259 | 0.164 | 0.462 | 0.557 |
| | [2.12] | [1.42] | [1.96] | [4.70] | [1.39] | [1.69] |
| $\mathrm{Beta}_{i,t}$ | 0.038 | -0.048 | 0.083 | 0.097 | -0.045 | -0.059 |
| | [0.56] | [-0.71] | [1.30] | [5.54] | [-0.96] | [-0.81] |
| $\log(\mathrm{Size}_{i,t})$ | -0.030 | -0.019 | -0.010 | -0.009 | -0.019 | -0.021 |
| | [-2.18] | [-1.46] | [-1.47] | [-5.24] | [-1.46] | [-1.61] |
| $\mathrm{B/M}_{i,t}$ | 0.071 | 0.068 | 0.003 | 0.001 | 0.068 | 0.069 |
| | [2.08] | [1.79] | [0.27] | [0.21] | [1.80] | [1.90] |
| $\mathrm{Ret}^{(12,1)}_{i,t}$ | -0.049 | -0.005 | -0.046 | -0.026 | -0.003 | -0.023 |
| | [-0.77] | [-0.09] | [-1.09] | [-1.66] | [-0.06] | [-0.40] |
| $\mathrm{SVIX}^2_t$ | | 2.793 | | | | |
| | | [1.90] | | | | |
| $\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t$ | | 0.513 | | | | |
| | | [1.44] | | | | |
| adj-$R^2$ (%) | 1.926 | 5.285 | 17.253 | 30.373 | 0.975 | 1.200 |
| $H_0: b_i = 0, c_i = 0$ | 0.003 | 0.001 | 0.701 | 0.000 | 0.188 | 0.091 |
| $H_0: b_i = 0, c_0 = 1, c_1 = 0.5$ | | 0.142 | | | | |

Table IA.6: Portfolio performance with monthly rebalancing ($\theta = 1$)

This Table summarizes the performance of monthly rebalanced portfolios invested in S&P 500 stocks from January 1996 to October 2014. We compare the performance of a portfolio based on model-implied expected excess returns to the market benchmark (a value-weighted portfolio) and a naive diversification strategy (an equally-weighted portfolio). For the model portfolio, we apply a rank weighting scheme,

$$w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t \, R_{i,t+1}]^\theta}{\sum_i \text{rank}[\mathbb{E}_t \, R_{i,t+1}]^\theta},$$

which assigns portfolio weights $w_{i,t}^{XS}$ that increase in expected excess returns, does not allow for short positions, and ensures that the investor is fully invested in the stock market, i.e. $\sum_i w_{i,t}^{XS} = 1$. The parameter $\theta$ controls the aggressiveness of the strategy; we set $\theta = 1$. For the model forecast-based portfolios, we report annualized average excess returns along with their associated standard deviations and Sharpe ratios as well as their skewness and excess kurtosis. For the benchmark portfolios we additionally report the performance fee (following Fleming et al., 2001) that a risk-averse investor with relative risk aversion $\rho \in \{1, 3, 10\}$ would be willing to pay to switch from the benchmark strategy to the model portfolio. The column labels indicate the variance horizon used in the model forecast.

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Portfolio performance based on model forecasts | | | | |
| Mean (%*p.a.*) | 10.48 | 11.03 | 11.06 | 11.45 |
| Sd (%*p.a.*) | 21.70 | 21.74 | 21.79 | 21.81 |
| Sharpe ratio (*p.a.*) | 0.48 | 0.51 | 0.51 | 0.52 |
| Skewness | −0.25 | −0.21 | −0.22 | −0.21 |
| Excess kurtosis | 1.61 | 1.54 | 1.60 | 1.51 |
| Comparison to value-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.45 | 0.44 | 0.43 | 0.41 |
| Skewness | −0.62 | −0.60 | −0.59 | −0.58 |
| Excess kurtosis | 0.83 | 0.77 | 0.74 | 0.68 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 3.02 | 3.61 | 3.81 | 4.44 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 2.73 | 3.31 | 3.51 | 4.15 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 2.54 | 3.13 | 3.32 | 3.97 |
| Comparison to equally-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.54 | 0.56 | 0.55 | 0.55 |
| Skewness | −0.48 | −0.46 | −0.46 | −0.45 |
| Excess kurtosis | 1.64 | 1.59 | 1.63 | 1.55 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 0.61 | 0.89 | 1.01 | 1.38 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 0.40 | 0.68 | 0.79 | 1.17 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 0.27 | 0.54 | 0.66 | 1.03 |

Table IA.7: Portfolio performance with daily rebalancing ($\theta = 1$)

This Table summarizes the performance of daily rebalanced portfolios invested in S&P 500 stocks from January 1996 to October 2014. We compare the performance of a portfolio based on model-implied expected excess returns to the market benchmark (a value-weighted portfolio) and a naive diversification strategy (an equally-weighted portfolio). For the model portfolio, we apply a rank weighting scheme,

$$w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t\, R_{i,t+1}]^\theta}{\sum_i \text{rank}[\mathbb{E}_t\, R_{i,t+1}]^\theta},$$

which assigns portfolio weights $w_{i,t}^{XS}$ that increase in expected excess returns, does not allow for short positions, and ensures that the investor is fully invested in the stock market, i.e. $\sum_i w_{i,t}^{XS} = 1$. The parameter $\theta$ controls the aggressiveness of the strategy; we set $\theta = 1$. For the model forecast-based portfolios, we report annualized average excess returns along with their associated standard deviations and Sharpe ratios as well as their skewness and excess kurtosis. For the benchmark portfolios we additionally report the performance fee (following Fleming et al., 2001) that a risk-averse investor with relative risk aversion $\rho \in \{1, 3, 10\}$ would be willing to pay to switch from the benchmark strategy to the model portfolio. The column labels indicate the variance horizon used in the model forecast.

| Horizon | 30 days | 91 days | 182 days | 365 days |
|---|---|---|---|---|
| Portfolio performance based on model forecasts | | | | |
| Mean (%*p.a.*) | 12.68 | 12.48 | 12.40 | 12.48 |
| Sd (%*p.a.*) | 24.89 | 24.94 | 24.93 | 24.83 |
| Sharpe ratio (*p.a.*) | 0.51 | 0.50 | 0.50 | 0.50 |
| Skewness | −0.11 | −0.11 | −0.12 | −0.12 |
| Excess kurtosis | 7.48 | 7.37 | 7.32 | 7.27 |
| Comparison to value-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.38 | 0.38 | 0.38 | 0.38 |
| Skewness | −0.18 | −0.18 | −0.18 | −0.18 |
| Excess kurtosis | 6.22 | 6.21 | 6.22 | 6.22 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 4.60 | 4.40 | 4.33 | 4.43 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 4.31 | 4.11 | 4.04 | 4.14 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 4.12 | 3.92 | 3.85 | 3.95 |
| Comparison to equally-weighted portfolio | | | | |
| Sharpe ratio (*p.a.*) | 0.51 | 0.51 | 0.51 | 0.52 |
| Skewness | −0.21 | −0.21 | −0.21 | −0.21 |
| Excess kurtosis | 7.29 | 7.29 | 7.29 | 7.30 |
| Performance fee with $\rho = 1$ (%*p.a.*) | 1.61 | 1.41 | 1.34 | 1.38 |
| Performance fee with $\rho = 3$ (%*p.a.*) | 1.35 | 1.16 | 1.09 | 1.14 |
| Performance fee with $\rho = 10$ (%*p.a.*) | 1.21 | 1.01 | 0.93 | 0.99 |

Figure IA.1: Beta, size, value, momentum, and option-implied equity variance

This Figure reports (equally-weighted) averages of risk-neutral stock variance ($\mathrm{SVIX}^2_{i,t}$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. At every date $t$, we assign stocks to decile portfolios based on on their characteristics and report the time-series averages of $\mathrm{SVIX}^2_{i,t}$ across deciles. The horizon is one month. The sample period is January 1996 to October 2014.

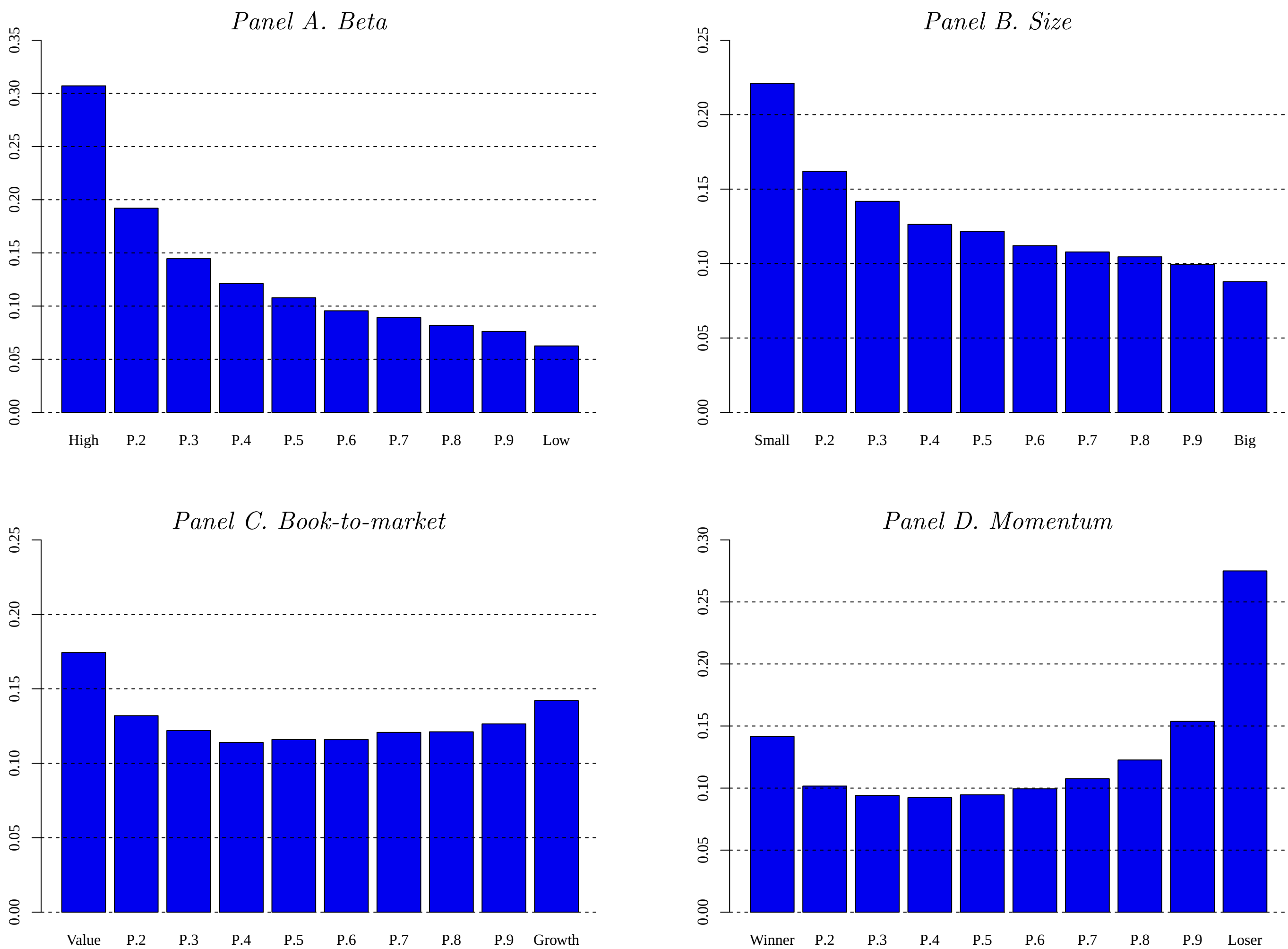


Figure IA.2: Beta, size, value, momentum, and option-implied equity variance

This Figure plots the time-series of risk-neutral stock variance ($\text{SVIX}^2_{i,t}$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. At each date $t$, we classify firms as small, medium, or big when their market capitalization is in the bottom, middle, or top tertile of the time-$t$ distribution across all firms in our sample, and compute the (equally-weighted) average of $\text{SVIX}^2_{i,t}$. We classify firms by other characteristics at time $t$ in a similar way. The horizon is monthly. The sample period is from January 1996 to October 2014.

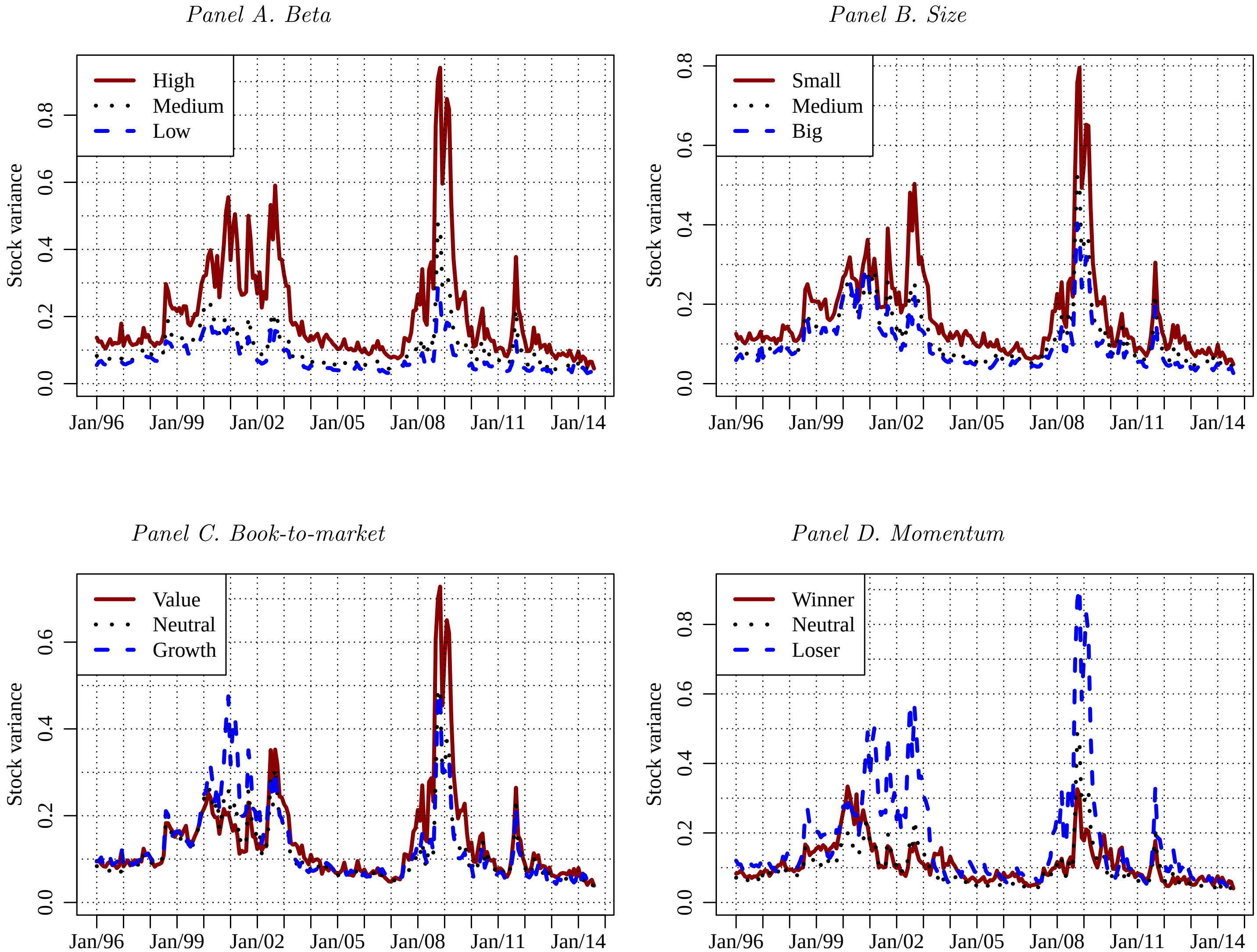


Figure IA.3: Size, value, and option-implied equity variance

This Figure plots the time-series of risk-neutral stock variance ($\text{SVIX}^2_{i,t}$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm size and book-to-market. At each date $t$, we classify firms as small, medium, or big when their market capitalization is in the bottom, middle, or top tertile of the time-$t$ distribution across all firms in our sample, and compute the (equally-weighted) average of $\text{SVIX}^2_{i,t}$. Similarly, we classify firms as value, neutral, or growth stocks when their book-to-market ratio is within the top, middle, or bottom tertile of the book-to-market distribution at time $t$. Panels A and B plot the time-series of $\text{SVIX}^2_{i,t}$-averages for intersections of size and value tertiles. The horizon is annual. The sample period is from January 1996 to October 2014.

*Panel A. Small stocks* *Panel B. Big stocks*

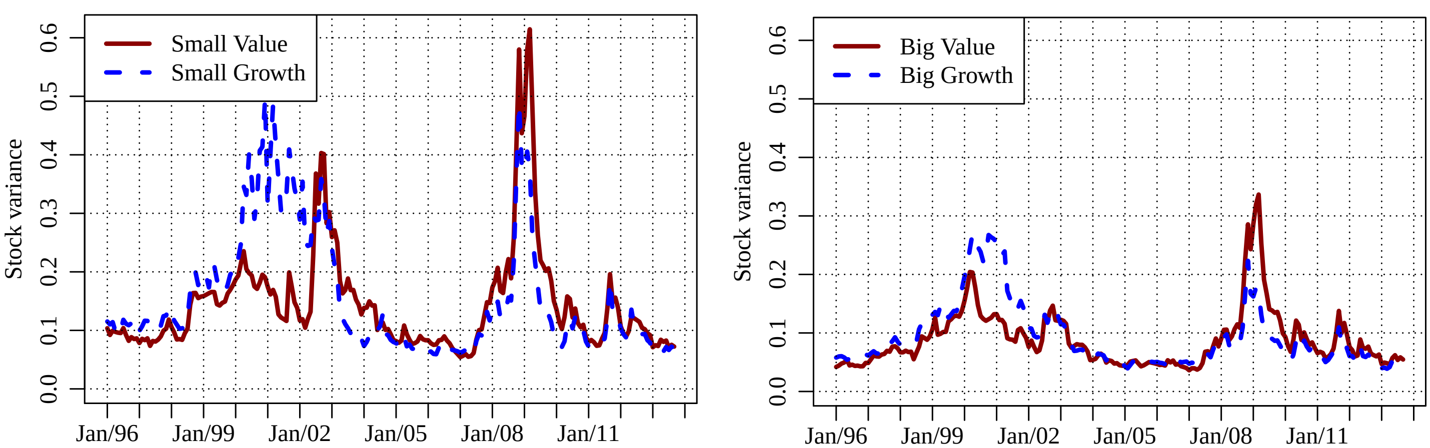


Figure IA.4: Portfolios sorted by size and book-to-market

This Figure presents results on the relation between equity portfolio returns in excess of the market and risk-neutral stock variance measured relative to stocks' average risk-neutral variance. At the end of each month, we form 25 portfolios based on a 5x5 double sort on size and book-to-market. For each portfolio, we compute the time-series average return in excess of the market and plot the pairwise combinations with the corresponding stock variance estimate multiplied by 0.5. The black line represents the regression fit to the portfolio observations with slope coefficient and R-squared reported in the plot legend. Our theory implies that the slope coefficient of this regression should be one and that the intercept corresponds to the average excess return of the market. The horizon is annual. The sample period is January 1996 to October 2014.

*25 Size x B/M portfolios*